

A New Escape Routing Strategy for Controlling Evacuation from Buildings

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Abstract—This paper proposes an approach for managing crowd evacuation from buildings. The work is motivated by the consideration that chaotic collective motion is a main reason for injuries and deaths during egress in emergency situations. Egress performance can be improved by real time adaptation of escape routes depending on feedback from the real scenario, considering crowd distribution and occurrence of random events. The proposed routing strategy aims at reducing the total egress time by driving individuals towards less congested exits, and is based on the comparison between the predicted evacuation time from each exit. Simulation performed on a simple case study by a commercial software illustrates the feasibility of the approach.

I. INTRODUCTION

The problem of safe evacuation of people in both standard and emergency conditions has been investigated by many researchers in recent years [1], [2], [3], [4], after impressive accidents and disasters. In particular, see the problem of minimizing the time for evacuation from a building to reduce the risk associated with an hazard (e.g. the exposure to gases and heat in the case of fire). Building evacuation models mainly lie in two classes [5], [6]. The first constitutes the network models, derived by an off-line procedure, which assumes that the occupants will not be affected by any stress and will follow exactly the instructions from some source of guidance, from which they are assumed to be totally controllable [7], [8]. These models are for the evacuation procedures based on static plans, which are fixed in advance by standard norms and regulation. Escape routes and time to safely egress all people are then established but no particular behavior (panic, psychological reactions or sociological links) or condition (age, sex, handicap, etc.) is considered to describe individual reactions and crowd motion. However, many people died or got injured because of panic and chaotic crowd movement where a better evacuation strategy could have saved many lives. A second class is then constituted by the behavioral models [9], based on individual attributes. In [8] the authors argue that most of existing methods does not deal with the negative effects of stressed crowds, then not optimal routing of people. They develop a method to find the optimal path, by predicting possible blocking. Other literature highlights the need for some guidance of panicked situations, where

any form of management or signal is useless. A decision-making process based on the emotional state of people is rather preferred to mathematical models [10].

Nowadays, new methodologies are available to design buildings and egress procedures, and modern technologies can be used to improve and speed-up the control of collective motion in evacuation. In particular, wireless networks of sensors and actuators can be profitably used to collect feedback from the environment and transmit simple but effective commands/indications to the people, in particular to expert agents helping the egress. Sensors could be not only the classical fire, smoke, and toxic gases (NO_x , CO, CO_2 , etc.) detectors, but also devices (e.g. cameras) developed to count and monitor people and foresee panic phenomena. In particular, feedback is necessary about: the distribution of individuals in the different parts of the environment (e.g. rooms of a building); the availability of doors, exit points and transit ways; flows in critical points; detection of overcrowding effects and blocking, congestion and deadlock conditions, especially close to exit points, doors and transit points.

It should be then possible to conceive and implement control strategies, which take into account different scenarios and individual characteristics to dynamically change escape routes, even if random and unexpected events occur (overcrowding, blocking of doors, structural collapses, congestion, etc.). Control should minimize the total egress time, because not so much can be gained to minimize the time necessary to recognize emergency and to process data from the scenario.

In this paper, a routing strategy is developed to support feedback control, which uses information from environmental data. Namely, a systematic method is defined to identify the state and speed of the egress procedure. All states are classified as safe (green), alert (yellow), and danger (red), according to the dynamic rate of change of the number of people occupying and leaving a certain space in a building, and taking into account the number of people and the time to evacuate, along with the characteristics of the evacuated space. This identification method allows us to on-line estimate three fundamental variables, namely the rate of evacuation, the time necessary for evacuation, and the final number of evacuated people. These predictions and the contemporaneous tracking of the current number of evacuated people helps us to define a control strategy that re-directs the crowd and minimizes the evacuation time. Control is based on simple rules affecting the messages displayed on monitors or information commanding other actuators. The

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$t_{d,i}$, since its gradient has a $C_{opt,i}$ value. The slope of m represents the average gradient $\alpha_{m,i}$ to evacuate P_i occupants in t_a seconds:

$$\alpha_{m,i} = \frac{P_i}{t_a - t_{d,i}} \quad (4)$$

The intersection between the o and a lines provides the maximum cumulative number of people $P_{max,i}$ evacuating through the exit in the time t_a :

$$P_{max,i} = C_{opt,i} \cdot (t_a - t_{d,i}) \quad (5)$$

If t_a and $C_{opt,i}$ are fixed, $P_{max,i}$ decreases with increasing values of $t_{d,i}$.

- 2) Yellow region (triangle ABD). Egress is evolving in an alerted manner. This region is limited by m , c lines and the time axis, where c represents the optimal egress starting at $t_{c,i}$:

$$C_{opt,i} = \frac{P_i}{t_a - t_{c,i}} \Leftrightarrow t_{c,i} = t_a - \frac{P_i}{C_{opt,i}} \quad (6)$$

A number P_i of people can always be evacuated in t_a seconds if $t < t_{c,i}$, possibly by some strategic action to increase the slope of the function $P(t)$. If $w_i = 1$, and therefore $C_{opt,i} = 2$, (6) can be reduced then to

$$t_{c,i} = t_a - \frac{P_i}{2} \quad (7)$$

where in this case:

- If $P_i = t_a$, it holds $t_{c,i} = t_a/2$ (see Fig. 2a).
- If $P_i = t_a/2$, it holds $t_{c,i} = (3/4)t_a$ (see Fig. 2b).

The width of the yellow region depends on the distance between $t_{c,i}$ and $t_{d,i}$, for $t_{d,i} < t_{c,i}$. When $t_{c,i} \leq t_{d,i}$ this region disappears. This is the case of a particular high $t_{d,i}$, due to a slow reaction time.

- 3) Red region (triangle BCD). Egress is evolving in a dangerous manner. This region is bounded by the a , c lines and the time axis. If egress evolves in this region, we calculate a new P_i value as reference. Normally, it has to be reduced, while being increased for some other exit, so that the state of the egress will here become green/yellow again.

To determine the state of the egress dynamics, the following relations can be applied. Note that for values $t \leq t_{c,i}$ and $t > t_{c,i}$ the conditions described below are related to different reference systems, respectively with origins in O and in D, as shown in Fig. 1. Thus:

- $t_{d,i} < t \leq t_{c,i}$: α is the gradient between segment AP and the t -axis

$$\alpha = \frac{P(t)}{t - t_{d,i}} \quad (8)$$

- $t_{c,i} < t \leq t_a$: α is the gradient between segment DP and the p' -axis.

$$\alpha = \frac{P_i - P(t)}{t_a - t} \quad (9)$$

With the above assumptions:

- 1) The state is green if

$$\begin{cases} \alpha_{m,i} \leq \alpha \leq C_{opt,i} & t_{d,i} < t \leq t_{c,i} \\ \alpha \leq \alpha_{m,i} & t_{c,i} < t \leq t_a \end{cases} \quad (10)$$

- 2) The state is yellow if

$$\begin{cases} \alpha < \alpha_{m,i} & t_{d,i} < t \leq t_{c,i} \\ \alpha_{m,i} < \alpha \leq C_{opt,i} & t_{c,i} < t \leq t_a \end{cases} \quad (11)$$

- 3) The state is red if

$$\alpha > C_{opt,i} \quad t_{c,i} < t \leq t_a \quad (12)$$

A relation between P_i and $P_{max,i}$ can now be found. From (5) and (6) it holds:

$$\frac{P_{max,i}}{P_i} = \frac{t_a - t_{d,i}}{t_a - t_{c,i}} \quad (13)$$

Therefore, two main cases can be described:

- 1) $P_i \leq P_{max,i}$. From (13) it holds that $t_{d,i} \leq t_{c,i}$. Therefore, since $t_{d,i} \geq 0$, from (6) it has to be also verified $t_a \geq \frac{P_i}{C_{opt,i}}$.
- 2) $P_i > P_{max,i}$. This condition is always verified for $t_a < \frac{P_i}{C_{opt,i}}$, and generally for $t_{d,i} > t_{c,i}$.

From (2) or (3), P_i can be large because of a large total number N of occupants or of a low availability of the number n of exits. Analogously, from (5), $P_{max,i}$ can be small due to a large $t_{d,i}$ or a small t_a values. In these cases, it's easier that the condition $P_i > P_{max,i}$ is verified.

The following extreme cases can be observed:

- 1) $t_{d,i} < t_{c,i}$, and therefore $P_i < P_{max,i}$, with $t_{c,i} = t_a$. The state can only be green/yellow. Note that from (6), this could be only verified for $P_i = 0$, which is not possible. Generally speaking, if $t_a \gg P_i$ it can be assumed $t_{c,i} \approx t_a$, and consequently the red region almost disappears, as shown in Fig. 2(c).
- 2) $t_{d,i} = t_{c,i}$, and therefore $P_i = P_{max,i}$. The green region is simply a line, from $t_{d,i}$, with gradient $C_{opt,i}$, whilst the remaining region is red (see Fig. 2(d)).
- 3) $t_{d,i} > t_{c,i}$, and therefore $P_i > P_{max,i}$. The state can only be red. In Fig. 2(e) this case has been shown with $t_{c,i} > 0$ and in Fig. 2(f) with $t_{c,i} < 0$.

III. PREDICTION OF THE FINAL STATE

Individual reactions to external stimuli are monitored by cameras. This information can be used by actuators for routing evacuees in a suitable way. However, the reaction of crowds to messages provided by actuators is not deterministic. For this reason, although the strategy described in the following allows an estimation of number of people to direct or re-direct towards the available exits, the instructions will be only provided in the form of colored symbols, to be more intuitive and immediate to read. The frequency of the message will depend on the level of danger. There will be a transient period, identified by the time $t_{c,i}$, where no particular message will be displayed. This is because,

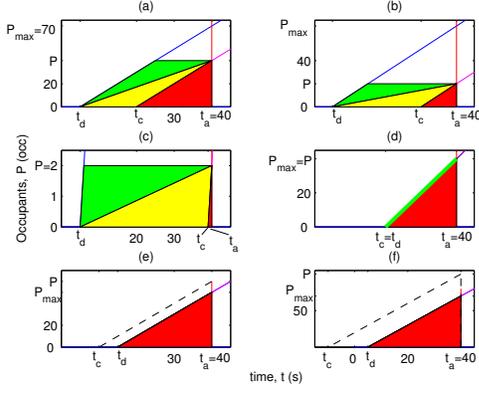


Fig. 2. A few particular cases of the egress dynamics with $C_{opt,i} = 2$

before t_c , it will always be possible to bring the evacuation dynamics back to a safe state. The set of rules will be then provided just for $t > t_{c,i}$.

The cumulative number of people $P(t)$ leaving an exit at the time t can be well approximated with a linear function:

$$P(t) = \alpha \cdot (t - t_{d,i}) \quad (14)$$

A linear relation is in fact also supposed by the software buildingEXODUS, where α is a chosen flow rate. If density of people increases, the flow rate decreases because of a penalty, applied by an algorithm, due to a number of conflicts among people. This penalty increases when people in conflict have the same drive to leave the door. The drive attribute is related also to the physical attributes. It's extremely unlikely that people with same drive attributes will conflict all time. As a consequence, it mostly holds a linear relations between t and $P(t)$. In our study, α can be estimated using the least-squares method (giving $\hat{\alpha}$) combined with a simple moving average, and will be denoted as $\alpha_{est}(t)$. The least-squares method will be applied for intervals of time. Thus, for each t , the average will be calculated using all the past values:

$$\alpha_{est}(t) = \frac{\sum_{k=1}^t \hat{\alpha}(k)}{t} \quad (15)$$

This gives a more reliable result than a simple least-squares applied to the whole period, particularly if $P(t)$ slowly varies within a large time interval. Ideally, it should be $P(t_a) \geq P_i$ and $\alpha_{est}(t_a) \geq \alpha_{m,i}$ since it's better to evacuate a number of people $P(t_a)$ larger than the target P_i and in a period of time smaller than the maximum available t_a . It is useful to analyze the trend of $P(t)$ for $t < t_a$, when a control strategy can improve the egress dynamics in terms of evacuation time. To this aim, the following quantities are defined:

- $ET_{est}(t)$: the estimated time needed to evacuate P_i occupants. From eq. (14)

$$ET_{est}(t) = \frac{P_i}{\alpha_{est}(t)} + t_{d,i} \quad (16)$$

- $P_{est}(t)$: the estimated cumulative number of people which can be evacuated at the time t_a . From eq. (14)

$$P_{est}(t) = \alpha_{est}(t) \cdot (t_a - t_{d,i}) \quad (17)$$

The characteristics of the prediction ($\alpha_{est}(t)$, $ET_{est}(t)$, $P_{est}(t)$) will be different for each instant t . Typically, starting from $t_{d,i}$, $P_{est}(t)$ increases from zero to an asymptotic value, possibly close to P_i , whilst $ET_{est}(t)$ decreases from infinite to an asymptotic value, possibly close to t_a . Ideally, the above mentioned asymptotic values should be equal to P_i and t_a or larger than P_i and smaller than t_a , as shown in Fig. 3.

Information about $P(t)$ are collected by means of sensors which could be damaged because of fire, gases, heat and vibration. In this case a message of warning is provided. The control system will include: cameras for counting people in the monitored area; an IEEE 802.15.4 technology based system for the radio localization of moving nodes to determine the position of people with appropriate ID.

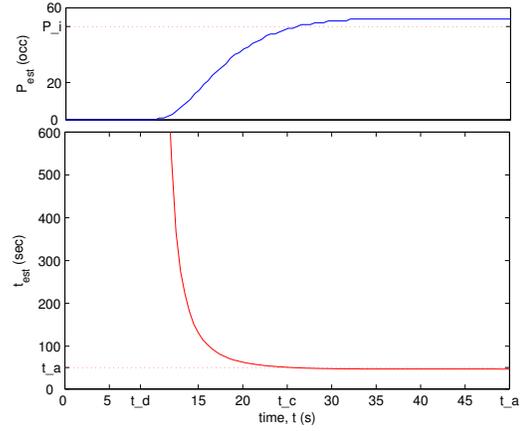


Fig. 3. The estimation characteristics of the egress dynamics

IV. THE CONTROL STRATEGY

A problem to choose if and where to re-direct the people arises. Here, the exits are all assumed to lead to the external environment. Let's denote with i and j two exits. An occupant in queue, waiting to exit from i , should compare and choose the smallest between the time to evacuate from i and j , denoted as t_i and t_j . Each of the above, are made up of two contributions: the time to wait in queue (D/C) and the time to reach that exit (d/v), where D is the density of people around the exit, C is the flow rate of that exit, d is the distance (m) between the occupant and the exit and finally v is the walking speed (m/s) of the occupant. An adaptive behavior will drive the occupant to the exit h verifying [15]:

$$\arg \min_h \left\{ \max \left(\frac{D_h}{C_h}, \frac{d_h}{v} \right) \middle| h = i, j \right\} \quad (18)$$

Therefore, the above behavior is implemented with an algorithm in buildingEXODUS v4.06 [15], when all the exits are visible, and therefore the occupants can re-direct depending on the level of congestion around the exits. However, it

is not likely that an occupant can clearly see all the exits (e.g. when smoke spreads all over) and he/she could also be unable to correctly choose, as the building EXODUS v4.06 supposes instead, the exit with a minimum evacuation time. Depending on the previously defined estimators, we aim to program a number of actuators to provide the crowd with some useful information, with the objective of minimizing the total evacuation time.

A. The escape routing rules

The variables previously defined allow us to build a number of main rules applied for $t > t_{c,i}$ to efficiently re-direct people.

- 1) If $t > t_{c,i}$ AND the state is green, no particular action to improve egress is required. If $D(t)$ is the density of people in a large area around the exit (so that it represents all the people who want to use that exit), two main issues arise:
 - If $\Delta P(t) = P_{est}(t) - P(t) > D(t)$: the exit will display a message with the number of people who can still come in the area around it, i.e. $\Delta P(t) - D(t)$. We don't expect that this number of people, that the actuator calculates, is the exact value, since it uses an estimation of the cumulative number of people. However, simulation results showed that the provided indication has a maximum error of three people, which is good.
 - If $\Delta P(t) = P_{est}(t) - P(t) < D(t)$: the exit will display a message with the number of people who have to leave the area around it, i.e. $D(t) - \Delta P(t)$. Again, this value has to be accepted with a certain margin of error.

The problem with a green egress state could be therefore linked to a partial use of the door potentiality or to a queue longer than the predicted maximum capacity.

- 2) If $t > t_{c,i}$ AND the state is yellow, the objective is to quickly bring back the state to green. From (4), (9) and (10), it holds that the required $P(t)$ to be in a green state is

$$P(t) > P_i \cdot \left(\frac{t - t_{d,i}}{t_a - t_{d,i}} \right) \triangleq P_G(t) \quad (19)$$

and therefore we aim at reaching a minimum additional number of people crossing the exit equal to $P_G(t) - P(t)$. There could be two reasons why a smaller number of people, with respect to its potentiality, is leaving through a certain exit. Each case will require a different action to try to make α grow:

- A low number of people is in the area. In this case α , and therefore $P(t)$, is low because nobody has left for long periods of time. This can be verified by analyzing $D(t)$ at a maximum distance of 2.5m from the exit. If the occupant is within this distance, he can not be redirected [15]. During the evacuation, except an initial transient phase, if $D(t)$ is small in the area within 2.5 m, it can be

assumed that it is small also for larger distances, and the actuators will have then to attract people.

- A very large number of people is in the area. If $D(t)$ is high (about the maximum capacity) and constant, α could be low because of a congestion. The actuator should therefore tell people to leave.

When a green state will be reached, the relative rules for this state will be applied.

- 3) If $t > t_{c,i}$ AND the state is red, a green state cannot be reached anymore, unless P_i is decreased. The exit will therefore display a message with the maximum number $P_i - P(t)$ of people who can stay in the area around it, with the aim of reaching:
 - A green state, defined by a new value P_i , by applying (19):

$$P_i < P(t) \cdot \left(\frac{t_a - t_{d,i}}{t - t_{d,i}} \right) \quad (20)$$

- A yellow state, defined by a new P_i value from (4), (9) and (11):

$$P(t) \left(\frac{t_a - t_d}{t - t_d} \right) < P_i < C_{opt,i}(t_a - t) + P(t) \quad (21)$$

When a green or a yellow state will be reached, the relative rules for these states will be applied.

Therefore, the choice to re-direct individuals or not is not based upon any adaptive behavior. Further, note that in this study we assume that each exit is not in danger (fire, smoke, crash), so that it is always safe to give people the instruction to re-direct towards to it. Finally, it is worth observing that the set of rules depends on the global state of all exits. Therefore, if a certain exit is too crowded when another one is barely used, the rules provided by actuators will be consistent to avoid oscillatory behaviors.

V. THE CASE STUDY

As case study, a single floor room with $n = 4$ exit doors of a unitary width ($w_i = 1, \forall i$) will be considered. The room has a rectangular $21 \cdot 14$ (m^2) plant (Fig. 4). A number of $N = 200$ young students (age between 19 and 30 years and an average walking speed between 1.2 and 1.5 m/s), are supposed to evacuate the room in a maximum time $t_a = 50s$. Building EXODUS v4.06 will be used for simulation, by setting attributes such that, if possible, during evacuation in normal conditions (i.e. without panic) people tend to keep the same direction of motion, with few changes in the angle of movement, and to avoid contacts, which reduces the population density. A *milling behavior* has also been supposed, to randomize the initial location of the occupants. Most legal regulations accept that the UFR exhibited by a system of evacuating people in normal conditions remains constant if the door width changes [17]. Some regulations adopt a specific flow rate of around 1.33 $occ/m \cdot s$. Here, the Fruin's distribution [14] has been used, since it covers a wider range of flow rates. Finally, an *adaptive behavior* will be switched on, to simulate the ability of people of judging

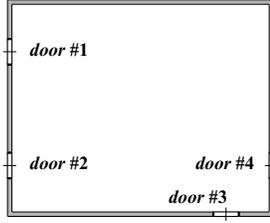


Fig. 4. Room layout

the best choice in terms of re-direction. The efficacy of the proposed control strategy will be verified for five different initial locations of people. Results will show that the strategy can in fact well predict if a certain door is too crowded or avoided, and therefore can provide for $t < t_a$ useful information about a more suitable egress. In this case study $t_{c,i} \triangleq t_c = 25s$ and $P_i = 50, \forall i = 1, \dots, n$.

The first distribution of people generated the results in Fig. 5.

5. The prediction built the following rules:

- Door 1: the actuator indicates to a number of people, decreasing from 14 to 7, to leave the area around the door. Simulation results showed that 7 queued people could not go through before t_a is reached (this can be seen counting the dots after t_a).
- Door 2: the actuator asks a number of people, decreasing from 16 to 2, to proceed towards the door. Simulation showed that from $t = 42s$ nobody has used the door, since for $t < t_a$ nobody was queuing anymore.
- Door 3: the actuator asks people to proceed towards the door. From $t = 38s$ nobody has crossed the door.
- Door 4: No particular requirement came from the actuators. Nobody queuing for this door has been left out from the evacuation and the people kept on exiting till t_a .

The second distribution of people generated the results in Fig. 6. The prediction built the following rules:

- Door 1: 3 people were attracted; nobody evacuated from $t = 47s$.
- Door 2: 9 people were attracted; nobody evacuated from $t = 42s$.
- Door 3: nobody evacuated from $t = 38.5s$.
- Door 4: 6 people invited to leave; 5 people not evacuated.

The third distribution of people generated the results in Fig. 7. The prediction built the following rules:

- Door 1: 5 people invited to leave; 4 people not evacuated.
- Door 2: 16 people were attracted; nobody evacuated from $t = 41s$.
- Door 3: nobody evacuated from $t = 37.5s$.
- Door 4: 5 people were invited to leave; 5 people not evacuated.

The fourth distribution of people generated the results in Fig. 8. The prediction built the following rules:

- Door 1: 1 person invited to leave; 1 person not evacuated.
- Door 2: 11 people were attracted; nobody evacuated from $t = 42.5s$.
- Door 3: nobody evacuated from $t = 39s$.
- Door 4: 3 people invited to leave; 3 people not evacuated.

The fifth distribution of people generated the results in Fig. 9. The prediction built the following rules:

- Door 1: 4 people invited to leave; 3 people not evacuated.
- Door 2: 13 people were attracted; nobody evacuated from $t = 44s$.
- Door 3: nobody evacuated from $t = 36.5s$.
- Door 4: 6 people invited to leave; 3 people not evacuated.

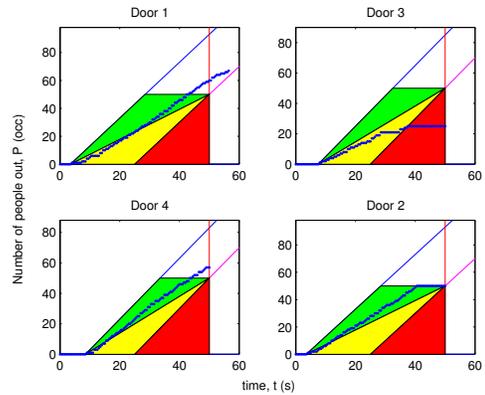


Fig. 5. Case 1: simulation results

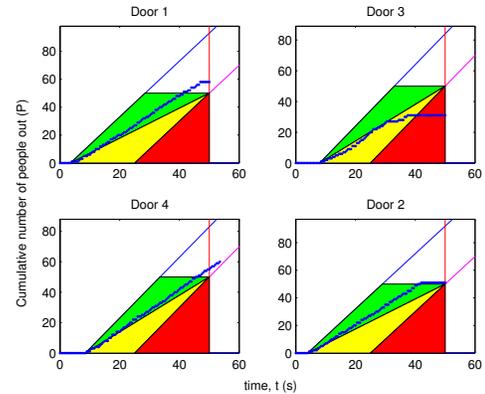


Fig. 6. Case 2: simulation results

VI. CONCLUSIONS AND FUTURE WORK

In this paper, a routing strategy for managing the crowd evacuation from buildings is proposed. Based on the prediction of left time to evacuate from each alternative exit, the strategy aims at reducing the total evacuation time by readdressing individuals to less congested paths. The routing strategy is conceived to be integrated in a supervisory control

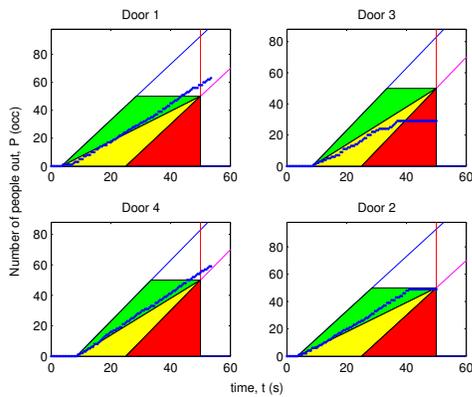


Fig. 7. Case 3: simulation results

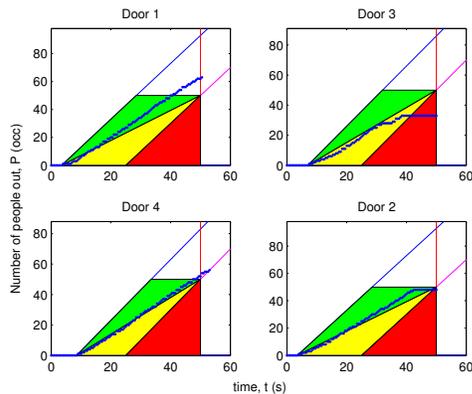


Fig. 8. Case 4: simulation results

system able to adapt the escape routes in real time by means of signs, sound messages and information given to operators. A simple case study shows the rationale behind the approach. Future work will regard the design and the implementation of a control strategy to drive the evacuation from complex buildings including alternative paths. More specifically, the proposed crowd routing approach will be applied to more complex buildings or scenarios by using modelling methodologies that could cope with complexity. For example, discrete event systems and queueing networks can be integrated to represent the dynamics of crowd during egress [18], [19].

VII. ACKNOWLEDGMENTS

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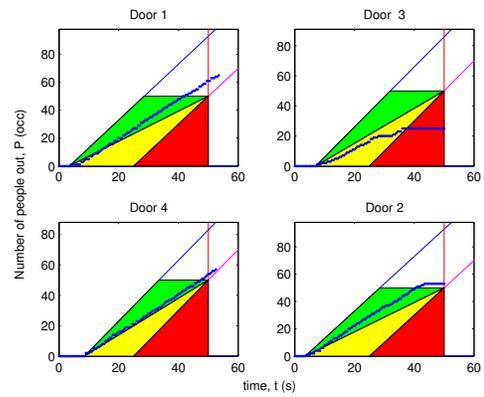


Fig. 9. Case 5: simulation results

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