A Linear Physical Programming Approach to Power Flow and Energy Storage Optimization in Smart Grids Models

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Keywords: Optimal Power Flow, Energy Storage, Smart Grids, Planning, Optimization, Uncertainties.

Abstract: The Optimal Power Flow problem (OPF) plays a crucial role in the successful energy management of modern smart grids. The diffusion of renewable energy sources poses new challenges to the power grid in which integrated energy storage combined with green generation solutions can help to address challenges associated with both power supply and demand variability. This work refers to a smart grid context and proposes a time indexed OPF model considering storage dynamics, adopting a preference-based optimization method with chance constraints to provide a suitable service level.

1 INTRODUCTION

Optimal Power Flow problem (OPF) plays a crucial role in the successful management of modern power grids.

The diffusion of renewable energy sources (even in the demand side) poses new challenges to the power grid in which integrated energy storage combined with green generation solutions can help to address challenges associated with both power supply and demand variability (Chandy et al., 2010, Koutsopoulos et al., 2011).

This paper reports on some optimization modeling results of a research work which refers to a smart grid context and proposes a time indexed Optimal Power Flow (OPF) model which considers storage dynamics and adopts a preference-based optimization method (on the generation side) joined with a chance constrained approach (on the demand side) to provide a suitable level of service.

2 PROBLEM DESCRIPTION

The OPF is a class of constrained optimization problems over a set of power/flow network variables (Carpentier, 1962). In general the variables may include active and reactive power outputs, generator or bus voltages and phases; while the objective may be the minimization of generation costs or the maximization of user utilities or level of service; and the constraints may be bounds on voltages or power levels, or that the line loading not exceeding thermal or stability limits. The OPF has been deeply studied during the last decades and several optimization techniques have been applied to both model and solve it (Dommel and Tinney, 1968, Kallrath et al., 2009).

In this paper, we develop a simple and general OPF model with energy storage and study how storage allows optimization of power generation across a given time horizon considering an uncertain demand and a system of preferences on the amount of generated power.

The proposed OPF approach belongs to the family of energy planning models (Huang et al., 2012, Khalid and Savkin, 2010) and aims to find a one-day-ahead energy production and distribution plan determining:

a) how much load (i.e. demand) to satisfy;
b) when and how much power to draw from the grid;
c) when and how to charge the energy storage system;
d) how to sell power back to the grid; while the goal is to minimize the overall costs including energy, devices and operations.

Besides OPF, which aims to search for the conditions which give the lowest cost for energy generation, storage and delivery, the implementation of the planning results may be based on different

Dellino G., Meloni C. and Mascolo S..
A Linear Physical Programming Approach to Power Flow and Energy Storage Optimization in Smart Grids Models.
DOI: 10.5220/0005293602240231
In *Proceedings of the International Conference on Operations Research and Enterprise Systems* (ICORES-2015), pages 224-231
ISBN: 978-989-758-075-8

actuation strategies including Demand Shaping (DS) and Energy Storage (ES).
The first is based on the (tentative) consumer demand shaping through financial incentives; encourages the consumer to: use less energy during peak hours and/or shift the time of energy use to off-peak times (i.e. night time, weekends). In the approach based on Energy storage (ES) ad-hoc units are required to store energy during off-peak hours and to discharge (i.e., supply energy) during peak hours (power leveling).

3 OPTIMAL POWER FLOW INCREMENTAL MODELING

Developing the Optimal Power Flow model we adopt an incremental approach interactively involving decision-makers (e.g., as suggested by Sierhuis and Selvin (1996)).
We start from a Conceptual Model (proposed by Chandy et al., 2010) as the basis to develop more complex and detailed models according to the needs and the practice of the context.

3.1 Conceptual Model

The Conceptual Model refers to a Single-Bus and Single-Generator case, but it can be extended to a network, i.e., Multi-Bus and Multi-Generator cases.

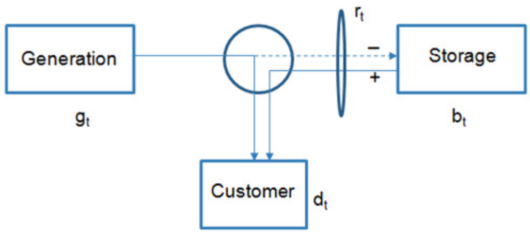


Figure 1: Reference scheme for the Conceptual Model with respect to a single node *i* of the grid.

The OPF Conceptual Model refers to the scheme depicted in Figure 1. It is a simple and general OPF model with energy storage and time-varying generation costs and power demand. It considers a single generator connected to a single load; the so called electric “per unit” DC model is assumed leading to a simplified structure of the network in which no reactive power is considered (Chandy et al., 2010). The main difference from the classical OPF is that storage allows optimization across time, e.g. charge when the cost of generation is low and discharge when it is high.

For each node i of the grid, the considered Conceptual Model is a time-indexed optimization problem characterized by a planning horizon containing T time slots t (i.e. $t = 1,...,T$) of the same length.
For each time slot t is known a value d_t for the power demand. The variables of the problem are the power g_t to be generate in time slot t, and the level of charge b_t of the storage system in the time slot t, which has a limited capacity of B. The energy flow to and from the storage system is indicated as r_t, i.e., assuming positive values while batteries are supplying energy and negative otherwise.
The considered optimization model includes a first set of demand satisfaction constraints, and set of constraints dealing with the level of charge of the batteries, while all variables are required to be non-negative.
The objective function is a cost to be minimized containing a generation and a storage component. The generation cost $c(g_t)$ can be assumed to be quadratic and (possibly) time-varying. The convexity of the cost function reflects a possible decreased efficiency when producing very high amounts of power (Chandy et al. 2010).
The storage cost $h(b_t)$ is assumed to be dependent only on the state of charge b_t (and not on the charge or discharge rate); it can be formulated as a linear penalty term for deviation from the desired target. An additional component $k(b_T)$ could be included to represent an optional penalty for the deviation from a final target value b_T (i.e., b_t with $t=T$) of the state of charge.
The overall formulation leads to the following mathematical program:

$$min\, Z = \sum_{t=1}^{T}[c(g_t) + h(b_t) + k(b_T)] \quad (1)$$

subject to (for all $t = 1,..., T$):

$$g_t + r_t = d_t \quad (2)$$

$$b_{t+1} = b_t - r_t \quad (3)$$

$$B - b_t \geq 0 \quad (4)$$

$$g_t \geq 0 \quad (5)$$

$$b_t \geq 0 \quad (6)$$

3.2 Case Study: Problem Setting and Preliminary Results

We consider, as “basic” case study, a problem introduced by Chandy et al. (2010) to illustrate the characteristics of the Conceptual Model reported in Section 3.1. This problem refers to a single-bus and

single-generator case under the assumption of an electrical “per unit” DC model.
The considered planning horizon is $T = 24$ hours and each time step t has a duration of 1 hour. According to the original model, the demand (in GJ units) profile in the planning horizon is represented by:

$$d_t = 50 + 10 \sin\left(\frac{4\pi\,(t-1)}{T-1}\right) \qquad (7)$$

The energy storage system is characterized by a battery capacity $B = 25$ GJ, and an initial level of charge $b_0 = 12.5$ GJ.
The additive components to the cost function Z –to be minimized– are:

$$c(g_t) = 0.5\,\gamma_t g_t^2 \qquad (8)$$

$$h(b_t) = a(B - b_t) \qquad (9)$$

As proposed by Chandy et al. (2010), in our “basic” case study some simplifications are included: i) the last component, related to the final level of charge of the batteries, is obmitted; ii) the cost coefficient is constant in the planning horizon and fixed to $\gamma_t = 1$ for all t (invariant case) While the value of the coefficient a was set to 2.

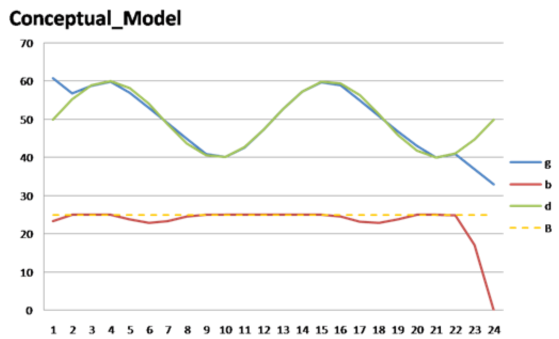


Figure 2: Conceptual Model Results for the basic case study. The x-axis reports the time slots, y-axis indicates power levels.

As depicted in Figure 2, according to Chandy et al. (2010), the optimal generation g_t is linear when the batteries charge and discharge, and follows the demand when the storage system is at the maximum level of charge. In the particular illustrative problem setting, the storage system is hardly used at all and appears quite oversized. In fact, it is mainly due to the relative values of generation and storage costs and their invariance in the planning horizon.
The energy storage system clearly needs a better modeling to deal with the level of charge at the end of the period. The decision-makers needs to consider in the planning activities also the preference and limitations on the generation-side and the possible uncertainties on the demand-side.

3.3 OPF: Enhanced Models

Starting from the Conceptual Model, and on the basis of the requirements defined in the application context, we are working, together with the decision-makers, on several modeling extensions mainly devoted to address the following issues, which compose our current research agenda:

1. Limitations on the flows to/from the storage system in each time slot;
2. Generated flow possibly assumes negative values (i.e., the Distribution System Operator (DSO) should receive energy from the node);
3. Demand predictions possibly assume negative values (i.e., customers should produce energy);
4. (Upper/Lower) Bounds on the amount of generated energy (possibly negative), e.g.,
   a) constant bounds;
   b) time-dependent (yet known) bounds;
   c) preferences (penalty based) on the level of power generation;
5. Uncertainties affecting the demand forecasts d_t;
6. Storage system inefficiencies with respect to holding, discharging and recharging phases;
7. Possible different energy sources;
8. More specific constraints related to the discharge and recharge phases of specific classes of energy storage systems;
9. Extension to a multi-generator and multi-bus context.

In particular, in this work we address the modeling issues related to points 4 and 5.

3.3.1 Uncertain Demand Forecasts

In general, uncertainties affecting the demand forecasts d_t are described as prediction intervals and error distributions (Box et al., 2008, Pflug and Römisch, 2007, Narayanaswamy et al., 2012, Conejo et al., 2010). On the basis of a demand forecast, the planner receives, for each time slot, the predicted value (i.e., d_t), the prediction interval, and the distribution of the values inside that interval.
On the basis of the forecasting values distribution within the prediction interval (e.g., from Autoregressive Integrated Moving Average

(ARIMA) or Support Vector Machines (SVM) time series models) we adopt a chance constrained approach (Charnes and Cooper, 1959) introducing a new set of constraints −to replace constraints (2) in the Conceptual Model (1)-(6)− in order to guarantee a given probability of demand satisfaction:

$$Pr[g_t + r_t \geq d_t] \geq 1\text{-}\alpha \ \ for\ \ t=1,...,T \qquad (10)$$

They represent a set of Level-of-Service (LOS) constraints and –noting that the uncertainty affects only the r.h.s of each constraint– can be linearized (Vanderbei, 2001) using the critical value d'_t associated to α (i.e., the specific required level of probability) through the probability density function of the demand:

$$g_t + r_t \geq d'_t \ \ for\ \ t=1,...,T \qquad (11)$$

Using these set of constraints instead of (2) leads to a first enhanced model, hereinafter indicated as EM1.

3.3.2 EM1: Additional Problem Settings and Results

To test the enhanced model EM1, the basic case study has been modified to consider the demand uncertainties. The forecasted demand value (i.e., the expected value) is assumed to be given by equation (7) for each time slot *t*.

The effect of prediction uncertainties has been modelled as a demand characterized in each time slot by a normal distribution (other distributions can be used instead) with mean given by the expected value d_t (i.e., through equation (7)). Two different scenarios has been considered. The first is characterized, in each time slot, by a standard deviation $\sigma_{5\%}=0.05d_t$, while the second by $\sigma_{10\%}=0.1d_t$. The probability of demand satisfaction is set to obtain 3 scenarios for the level of service (LOS): *70%*; *80%* and *90%*, respectively (according to equation (10)).

As an example, Figure 3 reports the results obtained by EM1 in the scenario characterized by a *LOS* of *90%* and the higher demand variability.

Due to more severe demand requirements, the use of the storage system increases in the optimal plan. The optimal generation g_t holds an almost linear behavior when the storage charges and discharges, and still matches the demand when the storage system is saturated. The storage system still calls for a better consideration of the behavior at end of the period.

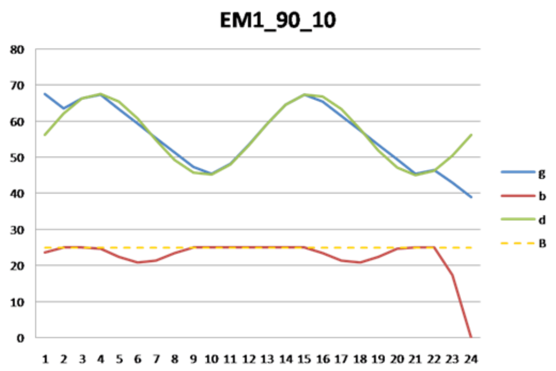


Figure 3: EM1 Results for the case study with *LOS = 90%* and $\sigma_{10\%}$. The *x*-axis reports the time slots, *y*-axis indicates power levels.

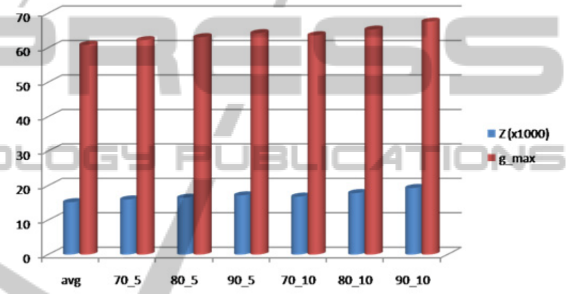


Figure 4: EM1 results for all scenarios: costs *Z* and the maximum generated power *g_max*.

Decision-makers acknowledge the practical relevance of this approach on the demand-side but they need to improve the model on the generation-side to allow a better management of the amount of generated energy.

More specifically, decision-makers find this models not comfortable as the *g_max* is close (or even over) the generation capacity of the node, and they are called to negotiate additional power to/from other nodes of the grid.

Figure 4 shows the results for all the scenarios in terms of the total cost *Z* and the maximum generated power *g_max* in the time slots within the planning period. In this figure, the scenario indicated as “avg” represents the base original scenario in which the expected values of the demand d_t are considered for each time slot (i.e., without demand variability). Other different scenarios are indicated in the x-axis with a label XX_Y, where XX represents the required LOS, and Y the amount of demand variability (i.e., $\sigma_{5\%}$ or $\sigma_{10\%}$).

3.3.3 A Preference-based Generation Management

In their planning activity, decision makers are subject to limitations in the amount of power that can be generated. Clearly, this issue can be easily addressed introducing a direct (and constant) bound on all the g_t variables.

More in general, we can consider the case with time-dependent (yet known) bounds in which, for each node of the grid, we introduce a capacity G_t^{MAX} that bounds the value of g_t. Nevertheless, decision makers are used to reason in terms of operative ranges, an approach which is only partially supported by a quadratic model for the generation costs as considered in the conceptual model and in EM1.

The need expressed by decision makers suggest us to setup a new enhanced model EM2 including the generation management with preferences on the different possible operative ranges.

In this modeling extension, these preferences are taken into account by a (linear) progressive penalty system.

The operative ranges in the feasible range $[0, G_t^{MAX}]$ indicated by the decision makers are represented in Figure 5 and are characterized as follows:

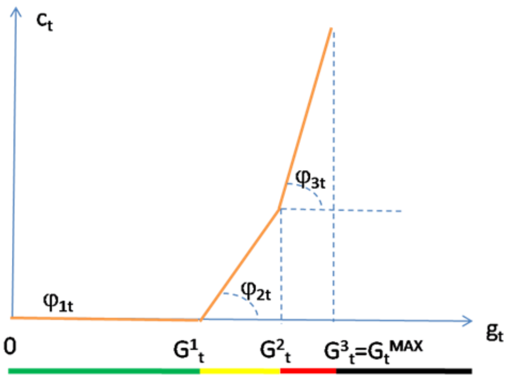


Figure 5: the generation cost c_t with a progressive linear penalty system based on different operative ranges.

OPERATIVE_RANGE 1 (OR1): $[0, G^1_t]$ this operational region is "preferred" or "desirable" and has an associated *NULL* penalty (in Figure 5 it is indicated in green) assuming $\varphi^1_t = 0$;

OPERATIVE_RANGE 2 (OR2): $[G^1_t, G^2_t]$ this region is considered "tolerable" (indicated in yellow in Figure 5), and has a penalty described by a slope $\varphi^2_t \geq 0$;

OPERATIVE_RANGE 3 (OR3): $[G^2_t, G^3_t]$ these values are "undesirable" (yet feasible, they are marked in red in Figure 5) and have a penalty represented by the slope φ^3_t, with $\varphi^3_t \geq 0$.

The level G^3_t coincides with G_t^{MAX} which indicates the production capacity of the system, i.e., any generation level $g_t > G_t^{MAX}$ is not feasible (in Figure 5 it is indicated in black).

For any feasible value g_t of energy generated in the time slot t, the cost is composed by a base-cost given by $c_t g_t$ and the additional penalty components depending on the region belonging g_t.

The system of penalties is formulated introducing a new set of constraints in the OPF model.

The first group of constraints takes into account the generation capacity in each time slot:

$$g_t \leq G_t^{MAX} \quad for\ t = 1, \dots, T \qquad (12)$$

For each ORi and for each time slot t, we introduce a new set of variables $\delta_{it} \geq 0$ representing the displacement in that operative range of the power generated during the time slot t. These displacement variables are required to satisfy, for each operative region ORi with $i \geq 2$, the following constraints:

$$g_t - \delta_{it} \leq G_t^{i-1} \quad for\ t = 1, \dots, T \qquad (13)$$

The additional (linear) contribute to the cost function for each time slot t is given by:

$$\sum_{i \geq 2} W_{it} \delta_{it} \qquad (14)$$

with $W_{it} = \varphi_{it} - \varphi_{i-1,t}$, and $\varphi_{1t} = 0$.

All these elements, in addition to the extensions already considered in EM1 lead to a new enhanced model we indicate hereinafter as EM2.

The weights W_{it} (and so slopes φ_{it}) are determined on the basis of additional indications provided by the decision makers.

More specifically, as the range limits define the preference internally to each single time slot (i.e., intra-period), they suggest a particular One-Versus-Other (OVO) rule to describe their inter-period preferences.

In fact, they prefer to minimize "as a priority" the number of "red" time-slots (i.e., those showing a positive displacement in *OR3*) and the amount of energy belonging in that region, and then those in "yellow" (i.e., the time slots limiting the power generation, at most, to *OR2*).

Overall, the preference based system proposed/shared with the decision makers belongs to the family of the so called Linear Physical Programming (LPP) Models (e.g., see Messac, 1996).

3.3.4 EM2: Additional Problem Setting and Results

To test the enhanced model EM2, the previous case study has been enriched. Besides the consideration of the demand uncertainties, the model of the energy storage system has been modified considering a battery capacity $B = 25$ GJ, a battery initial level of charge $b_0 = 0.8B$, and the following additional constraint on the final level of charge:

$$B_T \geq b_0 \tag{15}$$

and a new set of bounds on the minimum operative level of charge, required to be at least $b_{min} = 0.05B$:

$$b_t \geq b_{min} \quad for\ t = 1, \ldots, T \tag{16}$$

The component of the cost function related to the energy storage system is the same considered in EM1, while the base-component of the power generation cost is linear with a unitary cost given by $c_t = 0.5$, considered as constant in all the time slots. The components of the generation cost, related to the preference system and the OVO rule, are determined on the basis of the following *ORi*:

OR1: *[0, 50]*,

OR2: *[50, 57.5]*,

OR3: *[57.5, 60]*,

giving the weights $W_2 = 0.013$, and $W_3 = 1$.

The modeling of the demand behavior and its variability are the same considered in the setting of the previous case study, as well as the three *LOS* scenarios.

In general, due to the more challenge context, the use of the energy storage system has increased in all the considered scenarios playing an important role to cope with periods characterized by higher demand levels taking into account the generation constraints. It is worth to note that the linearity of the generated power when batteries are charging/discharging does not hold for EM2. Moreover, the storage system shows a satisfactory behaviour also in the terminal phase of the considered planning period.

Figure 6 reports a representative sample of the results obtained by EM2. In particular, the figure refers to the scenario characterized by a *LOS* of *80%* and the higher demand variability and clearly shows the power leveling effect of the energy storage system which charges during off-peak hours and discharges during peak hours.

Figure 7 shows the results for all the scenarios in terms of the total cost Z and the maximum generated power g_max in the time slots within the planning period. Again, as in Figure 4, the scenario indicated as “avg” represents the base original scenario without the superimposed demand variability, while other different scenarios are indicated with the same XX_Y notation. In this case, all the results are normalized w.r.t. the “avg” scenario and it is clear how EM2 is able to give an almost constant g_max among the different scenarios. As expected, these results show costs increasing as the LOS and the demand variability increase.

The decision makers, on the basis of the results obtained by the enhanced model EM2, often consider also different corrective actions including demand shaping (on the demand-side), negotiation to sell or buy energy (on the grid-side). EM2 gives useful information to support these management activities.

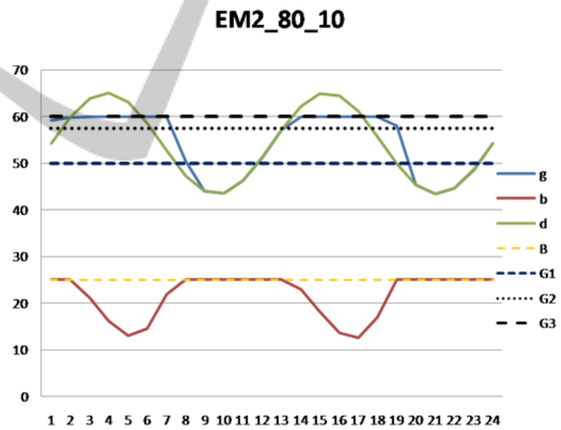


Figure 6: EM2 Results for the case study with $LOS = 80\%$ and $\sigma_{10\%}$. The *x*-axis reports the time slots, *y*-axis indicates power levels.

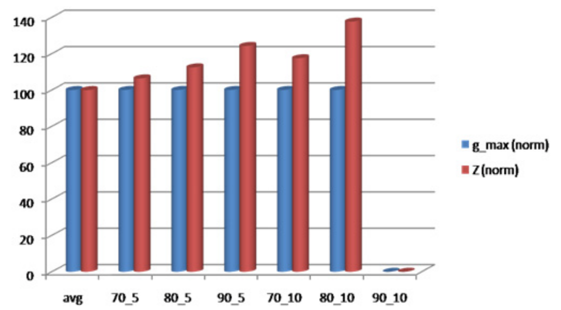
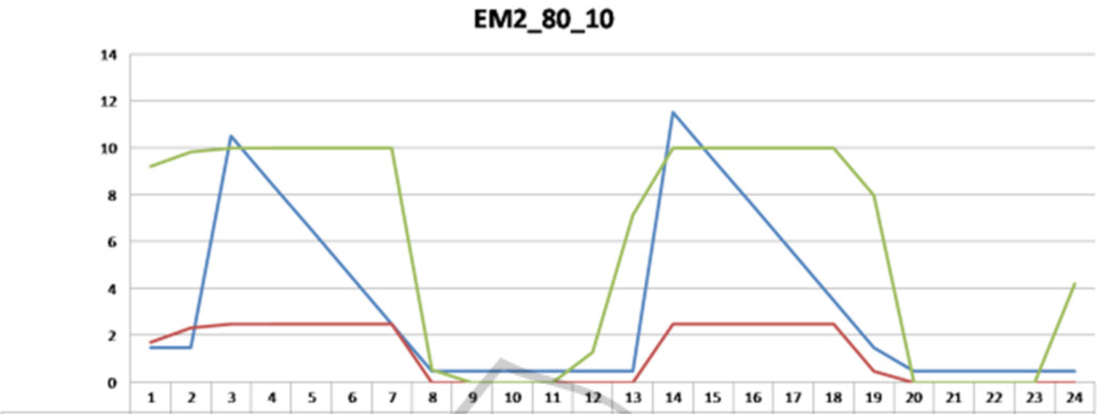


Figure 7: EM2 results (normalized w.r.t. the “avg” scenario) for all scenarios: costs *Z* and the maximum generated power *g_max*.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Shadow Price	1,51	1,51	10,5	8,51	6,51	4,51	2,51	0,51	0,5	0,5	0,5	0,51	0,51	11,5	9,51	7,51	5,51	3,51	1,51	0,5	0,5	0,5	0,5	0,51
delta3	1,70	2,34	2,5	2,5	2,5	2,5	2,5	0	0	0	0	0	0	2,5	2,5	2,5	2,5	2,5	0,48	0	0	0	0	0
delta2	9,20	9,84	10	10	10	10	10	0,54	0	0	0	1,28	7,13	10	10	10	10	10	7,98	0	0	0	0	4,20

Figure 8: EM2 Results for the case study with LOS = 80% and σ10%. Displacements variables in OR2 and OR3, and energy shadow prices in each time slot.

Firstly, these information are represented by the displacement variables in the different operative regions which indicate the level of production and its severity in each time slot.
Secondly, this information can be easily incorporated in a managerial dashboard joined with an estimation of the marginal value of a power unit (to sell or buy) in each time slot in the planning horizon.
EM2 provides this kind of estimation in terms of shadow prices associated to the demand (*LOS*) satisfaction constraints. Figure 8 reports −for the scenario with higher demand variability and a *LOS* of *80%*− an example of these useful information in numerical as well as graphical forms.

4 CONCLUSIONS

In this paper we develop an Optimal Power Flow model adopting an incremental approach interactively involving decision-makers.
We start from a simple conceptual model as the basis to develop more complex and detailed models according to the needs of the decision-makers trying to bridging the gap between modeling and the practice.
The interactive modeling development allows us to individuate several directions to develop enhanced models including the extension to networks (i.e., Multi-Bus and Multi-Generator cases); the representation of relevant storage system inefficiencies and more specific constraints related to the discharge and recharge phases of specific classes of energy storage systems.

ACKNOWLEDGEMENTS

This work was supported by the Project "RES Novae" (Reti, Edifici, Strade - nuovi obiettivi virtuosi per l'ambiente e l'energia), 2012-2015, funded by MIUR (Italy).

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