Tuning PI^{ν} Fractional Order Controllers for Position Control of DC-Servomotors

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***Abstract*—This paper introduces a loop shaping approach for easily tuning PI^{ν} fractional order controllers of position DC-servomotor drives. The design requirements are stated in terms of band-width and stability robustness. The synthesis procedure consists in shaping the open loop frequency response so as to achieve the flatness of its Bode's phase plot ensuring a good phase margin, which remains constant in a wide range around the crossover frequency. In this way, high stability robustness to gain variations in the loop is achieved. The fractional order integration also leads to limited overshoot and short settling time. Simulation results are confirmed by laboratory experiments.**

I. Introduction

According to recent surveys, PID (Proportional Integral Derivative) are the most used controllers in industrial applications, because of their tuning simplicity. They are employed, indeed, by more than 90% of feedback loops [1], [7]. On the other hand, in recent years, Fractional Order Controllers (FOC), based on calculus of non-integer order derivatives/integrals [17], have attracted much attention from the research community. The definition of the building block of FOC, however, dates back to the pioneering work of H. W. Bode in feedback amplifier design, which introduced the ideal transfer function $(s/\omega_{gc})^{\nu}$, where ω_{gc} is the gain crossover frequency and ν is the non-integer order [4]. Moreover, half a century ago, Tustin employed a FOC for controlling the position of massive objects [20] and, some years later, the papers [2], [14], [19] and the CRONE (i.e. *Command Robust d'Ordre Non Entier*) control [18] gave new impulse to the subject. From then on, FOC controllers have been used successfully in the industry of antenna, spacecraft and fire control systems. However, many design details have not been published because they were company property [15]. At present time, many researches focuses on tuning methods [3], [12], [13], [16] for taking full advantage of the flexibility characteristics of FOC. Namely, easily tuning approaches for FOC are still at their infancy.

This paper deals with tuning PI^{ν} for DC-servomotor drives, which are very important in many applications of mechatronics, robotics and motion control. More precisely, a new tuning approach is introduced for servomotors controlled by a PI^{ν}. The approach is based on the shaping of the open loop transfer function (TF), $G(j\omega)$, so that a nearly constant gain slope over a wide range frequencies around the 0-dB point is obtained. Hence, in accordance with the Bode phase-gain relation, the phase margin is maintained nearly constant in the same frequency interval and an excellent robustness to gain variations is obtained. The specifications on the open-loop TF are directly verified by simulation, to show how the changes of control parameters and, in particular, of the fractional order ν affect the performances and *viceversa*. Finally, laboratory experiments are compared with simulation results. The paper is organized as follows. Section II reviews some fractional calculus ideas. Section III introduces the proposed tuning approach. Section IV compares simulation and experimental results. Finally, Section V concludes the paper.

II. Overview of Fractional Systems

Historically, factional calculus originated from Riemann-Liouville definition of the fractional integral operator. Nowadays, the mathematical literature offers many different definitions of fractional derivatives and integrals. We refer to the Caputo's definition [5] of the fractional derivative of order ν of a function $f(t)$:

$$ {}_0D_t^{\nu} f(t) = \frac{1}{\Gamma(n-\nu)} \int_0^t (t-\tau)^{n-\nu-1} f(\tau) d\tau \qquad (1)$$

where $n-1 < \nu < n$, n is an integer and $\Gamma(\cdot)$ is the gamma function. Namely, the Caputo's definition is preferred here because its Laplace transform is given by:

$$\mathscr{L}\left\{{}_0D_t^{\nu} f(t)\right\} = s^{\nu} f(s) - \Sigma_{k=0}^{n-1} f^k(0) s^{\nu-k-1}, \qquad (2)$$

where the commonly used initial conditions appear. On the contrary, the Riemann-Liouville definition leads to initial conditions that do not have obvious physical interpretation. In (2), for $f^{(k)}(0) = 0$, $k = 0, 1, \ldots, (n-1)$, it holds: $\mathscr{L}\left\{{}_0D_t^{\nu} f(t)\right\} = s^{\nu} f(s)$. Even if (2) makes evident the meaning and potentialities of fractional order operators in applications, some difficulties arise because s^{ν} is irrational. However, there are many rational approximations to s^{ν} at disposal, see for example references in [11]. This paper refers to the following approximation given by one of the authors [10]:

$$s^{\nu} \cong \frac{\alpha_N(\nu, s)}{\beta_N(\nu, s)} \qquad (3)$$

where the denominator and numerator polynomials are both N-degree polynomials, with $N \geq 1$, whose coefficients depend on ν. More precisely:

$$\alpha_N(\nu, s) = \alpha_{N0}(\nu)s^N + \alpha_{N1}(\nu)s^{N-1} + \dots \alpha_{NN}(\nu) \quad (4)$$

$$\beta_N(\nu, s) = \beta_{N0}(\nu)s^N + \beta_{N1}(\nu)s^{N-1} + \dots \beta_{NN}(\nu) \quad (5)$$

Moreover, the coefficients of $\alpha_N(\nu, s)$ are given by:

$$\alpha_{Nj}(\nu) = (-1)^j B(N, j)(\nu + j + 1)_{(N-j)}(\nu - N)_{(j)} \quad (6)$$

where $B(N, j)$ is the binomial coefficient and where:

$$(\nu + j + 1)_{(N-j)} = (\nu + j + 1)(\nu + j + 2) \cdots (\nu + N) \quad (7)$$

$$(\nu - N)_{(j)} = (\nu - N)(\nu - N + 1) \cdots (\nu - N + j - 1) \quad (8)$$

define the Pochammer functions, with $(\nu + N + 1)_{(0)} = 1$ and $(\nu - N)_{(0)} = 1$. In addition, it holds: $\alpha_{Nj}(\nu) = \beta_{N,N-j}(\nu)$.

III. The Loop Shaping Method for Tuning PI^ν

With reference to a classical unitary feedback loop let:

$$G_C(s) = K_P + \frac{K_I}{s^\nu} = \frac{K_I(1 + T_C s^\nu)}{s^\nu} \quad (9)$$

with $T_C = K_P/K_I$ and $0 < \nu < 1$, be the TF of the PI^ν controller and:

$$G_P(s) = \frac{K_E}{s(1 + T_E s)} \quad (10)$$

be the DC-servomotor TF. If $G(s) = G_C(s)G_P(s)$, then the closed-loop TF, say $F(s)$, is given by:

$$F(s) = \frac{1}{1 + G^{-1}(s)}. \quad (11)$$

Now, replace s with $j\omega$ in (11). According to the seminal paper by Kalman [6] << *a feedback system is optimal if and only if the absolute value of the return difference is at least one at all frequencies* >>. Obviously, in physical systems, the magnitude of return difference $|1 + G^{-1}(j\omega)|$ cannot satisfy this condition, which, indeed, states that the reference signal, $r(j\omega)$, must equal the output, $y(j\omega)$, i.e. $r(j\omega) \equiv y(j\omega)$, for all frequencies. Of course, this condition cannot be exactly verified and must be approximated in a desired frequency band only. Therefore, to obtain a good tracking performance, it must be $|F(j\omega)| \cong 1$, which can be transformed in a corresponding constraint on the open loop TF, i.e. $|G^{-1}(j\omega)|^2 \ll 1$. To this aim, we employ (9) and (10) to give:

$$G^{-1}(j\omega) = \frac{\omega^{1+\nu}\{\cos[\vartheta(\nu)] + j\sin[\vartheta(\nu)]\}(1 + j\omega T_E)}{K_E K_I\{1 + \omega^\nu T_C[\cos(\theta(\nu)) + j\sin(\theta(\nu))]\}} \quad (12)$$

where $\vartheta(\nu) = 0.5(1 + \nu)\pi$ and $\theta(\nu) = 0.5\nu\pi$. Since $|G^{-1}(j\omega)|^2 = G^{-1}(j\omega)G^{-1}(-j\omega)$, it follows:

$$|G^{-1}(j\omega)|^2 = \frac{\omega^{2(1+\nu)}(1 + \omega^2 T_E^2)}{(K_E K_I)^2\{1 + \omega^{2\nu}T_C^2 + 2\omega^\nu T_C cos[\theta(\nu)]\}} \quad (13)$$

Now, introducing the non-dimensional frequency $u = \omega T_E$, (12) gives:

$$G(ju) = \frac{K_E K_I\left\{1 + \left(\frac{u}{T_E}\right)^\nu T_C[cos(\theta(\nu)) + j\sin(\theta(\nu))]\right\}}{(\frac{u}{T_E})^{1+\nu}\left\{\cos[\vartheta(\nu)] + j\sin[\vartheta(\nu)]\right\}(1 + ju)} \quad (14)$$

Therefore, it holds:

$$\angle G(ju) = \tan^{-1}\left(\frac{\left(\frac{u}{T_E}\right)^\nu T_C\sin(\theta(\nu))}{1 + \left(\frac{u}{T_E}\right)^\nu T_C\cos(\theta(\nu))}\right) + -\vartheta(\nu) - \tan^{-1}(u) = \varphi_1(u) - \varphi_2 - \varphi_3(u). \quad (15)$$

Moreover, we may write:

$$|G^{-1}(ju)|^2 = A(u)B(u) \quad (16)$$

where

$$A(u) = \frac{1}{(K_E K_I)^2}\left(\frac{u}{T_E}\right)^{2(1+\nu)} \quad (17)$$

$$B(u) = \frac{1 + u^2}{1 + \left(\frac{u}{T_E}\right)^{2\nu}T_C^2 + 2\left(\frac{u}{T_E}\right)^\nu T_C cos(\theta(\nu))} \quad (18)$$

Note also that $A(u)$ and $B(u)$ depend on the unknowns K_I and T_C, respectively.

The design approach for choosing the controller parameters is as follows. Firstly, to respect the constraint $|F(ju)|^2 \cong 1$, the specifications are stated in what they demand of the open loop transmission, $G(ju)$. More precisely, T_C, K_I, and ν must be selected so as to respect $|G^{-1}(ju)|^2 \ll 1$ in a significant frequency range. This constraint is important to design positional servomechanism, where the instantaneous following or tracking error is an important specification. Secondly, as a guarantee of stable performance despite drive parameters changes, stability margins must be ensured by appropriately shaping of $G(ju)$ in the crossover region. To this aim, the proposed shaping approach takes advantage of features of the Bode plots of the fractional integrator, which shows a "flat" phase curve in a wide frequency interval and a magnitude plot with fractional slopes of $-20\nu dB/decade$.

To satisfy the first constraint, we choose the frequency band u_B so that $A(u_B)B(u_B) = \varepsilon$, where $\varepsilon > 0$ is a conveniently small number. Accordingly, we begin to set $T_C = \tilde{T}_C$ so that $B(u_B, \tilde{T}_C) < 1$, where $B(u_B, \tilde{T}_C)$ is the value of $B(u)$ given by (18), assumed for $u = u_B$ and $T_C = \tilde{T}_C$. This condition is certainly satisfied for $\tilde{T}_C$, such that:

$$\tilde{T}_C = a(u_B)^{(1-\nu)}T_E^\nu = aT_{c0} \quad (19)$$

where the constant $a > 1$ is to be chosen. Appropriate values for a, i.e. for $\tilde{T}_C$ and ν, are selected for obtaining a desired phase margin $M\phi$, which is held constant in wide interval around the $0 - dB$ crossover frequency u_C. On the other hand, u_C is usually very close to the closed loop bandwidth u_B and can be estimated by $u_C \approx u_B/1.3 \div u_B/1.7$ [8], [9] and hence we assume $u_C = u_B/1.7$ in the following. Thus, substituting

$u = u_C$ in (15) easily gives: $M\phi = \varphi_1(u_C) + 0.5(1-\nu)\pi - \varphi_3(u_C)$. Since, if $\varphi_1(u_C) = \varphi_3(u_C)$, it holds:

$$M\phi = 0.5(1-\nu)\pi, \tag{20}$$

this suggests us to choose $a = \tilde{a}$ so that (20) is satisfied. Some easy calculations yield:

$$\tilde{a} = \frac{1}{1.7^{1-\nu}\{\sin[\theta(\nu)] - u_C \cos[\theta(\nu)]\}} \tag{21}$$

Note that $\tilde{a}$ can ever be fixed so that $\tilde{a} > 1$. It can be also observed that the values of $\tilde{a}$ and hence of $\tilde{T}_C$ do not change significantly in the range around u_C. In practice, as it is shown in the nest section, $M\phi$ does not change significantly respect to the value given by (20).

At this point, we can determine the second parameter K_I by putting:

$$K_I = \frac{1}{K_E}\left(\frac{u_B}{T_E}\right)^{(1+\nu)}\sqrt{\frac{B(u_B,\tilde{T}_C)}{\varepsilon}} \tag{22}$$

which completes the tuning method.

IV. Simulation and Experimental Results

To validate the tuning approach, we have performed some simulation and laboratory experiments. The experimental set up consists of a nonlinear 370W DC servomotor (AMIRA DR300), a power amplifier driving the plant, and a PC equipped with a floating point 250Mhz Motorola PPC dSPACE board (DS1104), which provides the position reference and runs the controllers. All routines run in discrete time with a 1 ms sampling period. The DC motor transfer function has been obtained through a frequency domain identification process, yielding $K_E = 0.9779$ and $T_E = 0.0798s$ as gain and time constant of the servomotor. A 1024 pulses incremental encoder gives the rotor position measurement. We implemented the control algorithms in the MATLAB/Simulink environment. The dSPACE code generator compiles the Simulink program and then the real-time executable code is downloaded to the board memory. During motor operation, the board processor receives the feedback from the encoder and applies the appropriate control action to the power unit. The signals are processed using 16 bit A/D-D/A converters that are integrated in the dSPACE board.

We tuned the FOC controller with formulae (19) and (22), considering the following values: $u_B = 0.07$ (then $\omega_B = 0.877 rad/s$), $\varepsilon = 0.01$, and a varies with ν according to (21). Moreover, to analyze performances the irrational operator s^ν is approximated with a rational integer order TF given by 3 taking the fifth order N = 5. Using this approximation, Figures 1 and 2 depicts the Bode plots of $G(ju)$ and $F(ju)$, respectively, for $\nu = 0.2, 0.3, 0.4, 0.5, 0.6$. In particular, given $u_C = u_B/1.7 = 0.041$, Fig. 1 shows phase margins for different orders ν. Moreover, due to the "flatness" of the phase Bode-plots, $M\phi$ remains nearly unchanged if the gain varies around its nominal value.

Fig. 3 compares the step responses obtained by simulation (dashed line) and by measured data (continuous line), for $\nu =$ 0.2 to $\nu = 0.6$. The responses show a good agreement between simulations and laboratory experiments, with slight differences due to motor static nonlinearities mainly affecting the settling time. Experiments confirm that increasing ν results in a higher percentage overshoot, a lower rising time and a longer settling time.

Finally, experimental results in Figures 4, 5 and 6 point out how performance indexes are affected by the choice of u_B and ν. In particular, increasing u_B makes the system faster, reducing the rising time and the settling time, and increasing the percentage overshoot. Also, Fig. 4 shows that a proper choice of ν can lead to a low percentage overshoot combined

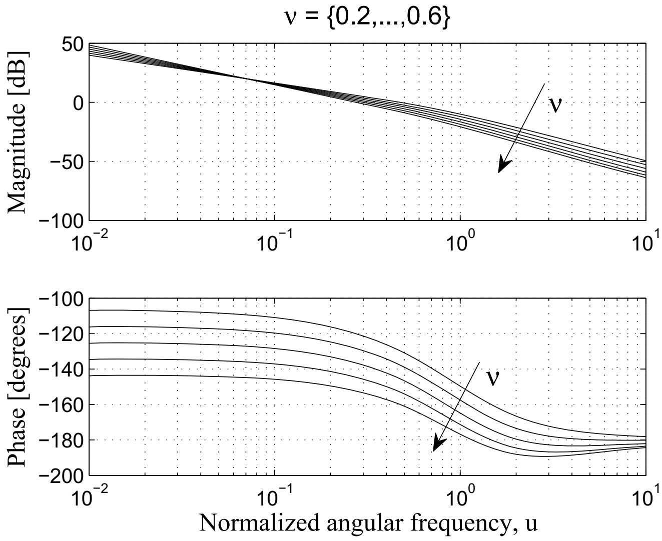

Fig. 1. Open-loop frequency responses for $u_B = 0.07 rad/s$ and different values of parameter ν.

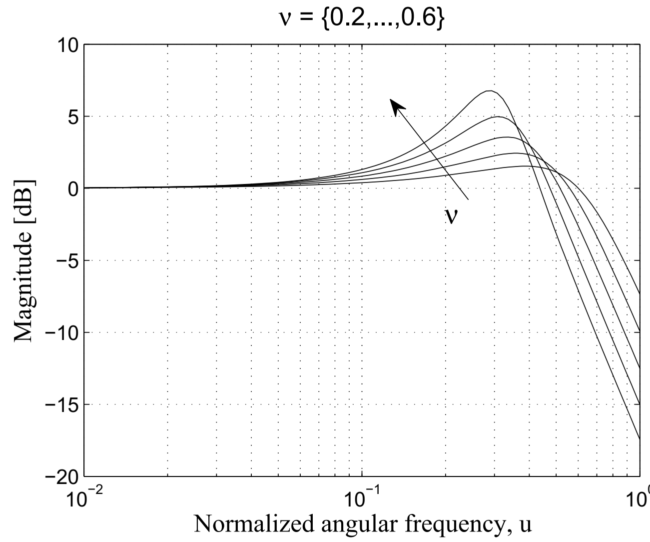

Fig. 2. Closed-loop frequency responses for $u_B = 0.07 rad/s$ and different values of parameter ν.

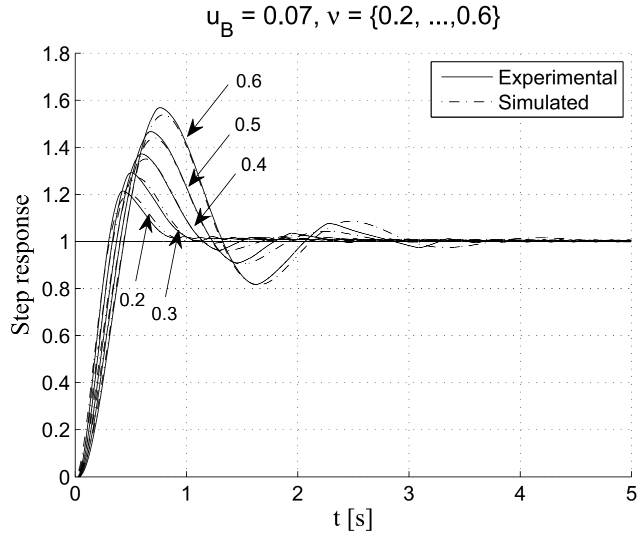

Fig. 3. Closed-loop step response of controlled system for $u_B = 0.07 rad/s$ and different values of parameter ν.

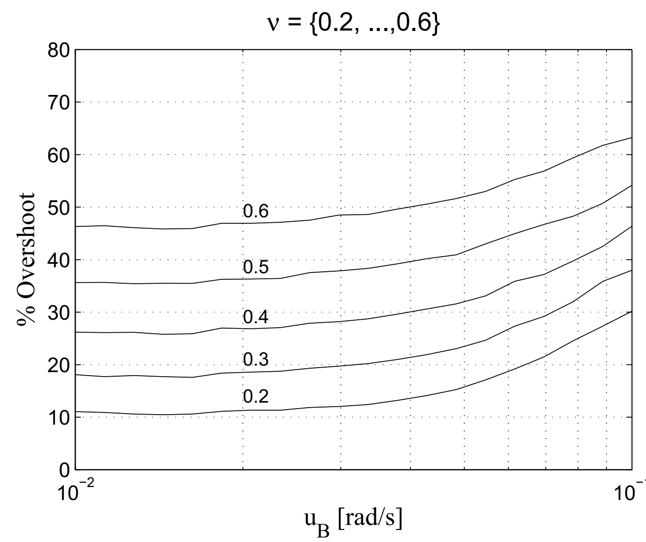

Fig. 4. Experimental percentage overshoot for different values of parameter ν.

with an excellent Phase Margin.

V. Conclusion

A new approach is proposed for tuning PI^{ν} position controllers of DC-servomotors. Given the desired bandwidth u_B, which is the main factor in determining the rise time, the proposed approach leads to the free parameters of the FOC controller (i.e. ν, T_C and K_I) making u_B compatible with a required phase-margin. In addition, despite gain variations, the phase margin remains unchanged in a wide frequency interval, because the phase Bode-plot of $G(j\omega)$ is nearly flat around the crossover frequency. Simulations and experiments are in good

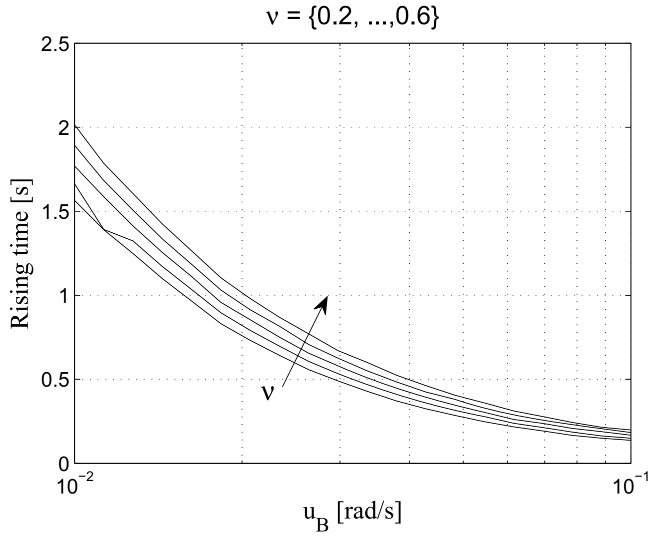

Fig. 5. Experimental rising time for different values of parameter ν.

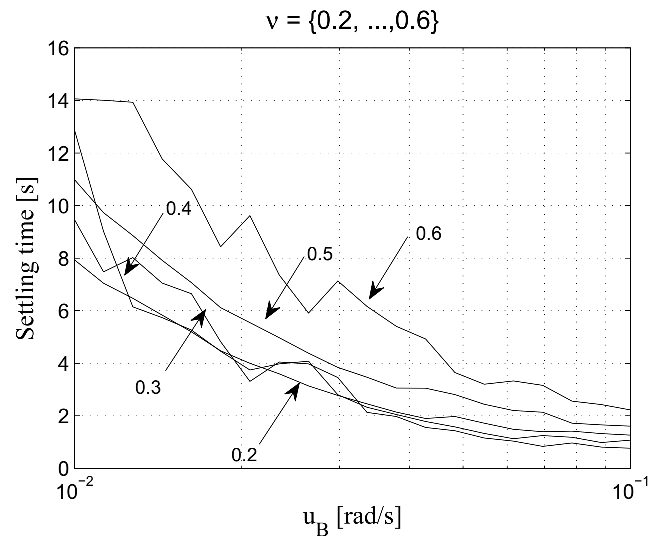

Fig. 6. Experimental settling time for different values of parameter ν.

agreement and show that appropriate selection of fractional order, ν, leads to good values for overshoot and settling times.

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