Synchronization of Hyperchaotic Cellular Neural Networks: a System Theory Approach

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Abstract

In recent years synchronization of chaotic dynamics has received ever increasing attention. Herein, Cellular Neural Networks (CNNs) are considered as a tool for generating hyperchaotic behaviors. By exploiting a system theory approach, a technique for synchronizing a large class of CNNs is developed. In particular, a necessary and sufficient condition for hyperchaos synchronization is given, which is based on the controllability property of linear systems. Finally, in order to show the effectiveness of the proposed technique, the synchronization of a CNN constituted by Chua's circuits is illustrated.

Keywords: Cellular Neural Networks, Neural Circuits, Chaos Synchronization, Neurodynamics.

1. Introduction

Synchronization of chaotic systems has been the subject of many papers over the last few years [1]-[5]. However, most of the developed methods concern the synchronization of low-dimensional systems, that is, chaotic systems with only one positive Lyapunov exponent. Since these systems are characterized by dynamics of limited complexity, the attention has been recently focused on higher dimensional chaotic systems. In fact, dynamic systems with several positive Lyapunov exponents exhibit more complex dynamics, which can be exploited for secure communications [6]-[9], pattern formation [10] and active wave propagation [11]

Regarding high-dimensional systems and their synchronization, the state of the art does not give a practical answer to the following important questions:

1) How can one obtain a high-dimensional system, which exhibits hyperchaotic attractors, starting from low-dimensional systems as building blocks?

2) Given high-dimensional systems, can synchronization be achieved in a systematic way?

The aim of this paper is to make a contribution in these directions. In our opinion, this high-dimensional challenge could be within reach by imposing some regularity and symmetry on the state space, which is the case in Cellular Neural Networks (CNNs). It is well known that CNNs are dynamic arrays of simple circuit units that can be easily implemented using VLSI technique [12]. By considering simple chaotic units as neural cells, the interconnections of a sufficiently large number of neurons can exhibit extremely complex behaviors, such as high-dimensional chaotic attractors [11]

Following these considerations, the idea underlying this paper is to consider CNNs as a tool for generating hyperchaotic behaviors. By exploiting a system theory approach, a technique for synchronizing a large class of high-dimensional system is developed. The proposed method is rigorous and *systematic,* that is, synchronization can be achieved if some structural properties related to the synchronization error system hold.

The paper is organized as follows. In section 2 the CNN architecture is briefly illustrated, whereas in section 3 basic notions concerning chaos synchronization are reported. In section 4 it is shown how two hyperchaotic CNNs can generate a linear and time-invariant synchronization error system. Then, by checking the controllability property of linear systems, a necessary and sufficient condition is given in order to synchronize the CNN dynamics. Finally, in section 5 the proposed method is applied to synchronize a hyperchaotic CNN constituted by Chua's circuits.

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2. Hyperchaotic CNNs

A CNN is a dynamic array of simple circuits particularly suitable for VLSI implementation [12]. The interconnections of a sufficiently large number of simple units can exhibit extremely complex behaviors, such as Turing patterns, spiral and scroll waves or high-dimensional chaotic attractors [11]. Since the attention is focused on hyperchaotic dynamics, the following definitions and assumptions are reported.

Definition I: An N-cell CNN is defined mathematically by four specifications [11]:

1) CNN cell dynamics;
2) CNN synaptic law;
3) Initial conditions;
4) Boundary conditions.

The basic CNN cell is shown in Fig 1. It contains, in addition to the dynamical circuit core characterized by its state vector $\boldsymbol{x}_i$, an input $\boldsymbol{u}_i$, a dc bias $\boldsymbol{z}_i$, an output $f(\boldsymbol{x}_i)$ and a synaptic input current $\boldsymbol{I}_i$, which describes the interactions among cells.

2.1 CNN cell dynamics

Assumption 1: Herein, the i-th cell is a dynamic circuit described by a set of M differential equations:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{C}\boldsymbol{x}_i + \boldsymbol{d}\ f(\boldsymbol{x}_i) + \boldsymbol{z} \quad \text{for } i = 1,\ldots,N \tag{1}$$

where $\boldsymbol{x}_i \in \Re^M$ is the state vector, $\boldsymbol{z} \in \Re^M$ is the dc bias, $C \in \Re^{M\times M}$ is a constant matrix, $\boldsymbol{d} \in \Re^M$ is a constant vector and $\boldsymbol{f}(\boldsymbol{x}_i)$ is the scalar output, with $\boldsymbol{f}: \Re^M \to \Re$.

Remark 1: Several chaotic circuits can be modeled by (1). For instance, Chua's circuit [11], the higher dimensional Chua's circuits [13], and the hyperchaotic circuits in [14]-[15] are examples of systems satisfying Assumption 1. Hence, they can be used as building blocks to obtain hyperchaotic CNNs.

2.2. CNN synaptic law

It is worth noting that the CNN synaptic law depends on the input and the state of all cells located within a prescribed *sphere of influence,* or *neighborhood size* [11]. Although the contribution from the input and the state of each neighbor cell may be any arbitrary nonlinear coupling, in the following linear interactions are considered.

Assumption 2: The synaptic law of the i-th CNN cell is:

$$\boldsymbol{I}_i = \boldsymbol{H}_i\left[\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \ldots, \boldsymbol{x}_N^T\right]^T \quad \textbf{for } i = 1,\ldots, \mathrm{N} \tag{2}$$

where $\boldsymbol{I}_i \in \Re^M$ is the synaptic current whereas $\boldsymbol{H}_i \in \Re^{M\times MN}$ is a sparse matrix that takes into account the *local connection* among cells.

By considering (1) and (2), the whole CNN becomes:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{C}\boldsymbol{x}_i + \boldsymbol{d}\ f(\boldsymbol{x}_i) + \boldsymbol{z} + \boldsymbol{I}_i \quad \textbf{for } i = 1,\ldots,N \tag{3}$$

In compact form, (3) can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{c} \tag{4}$$

where $\boldsymbol{x} = \left[\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \ldots, \boldsymbol{x}_N^T\right]^T \in \Re^{MN}$,
$A = \left(diag(\boldsymbol{C}) + \boldsymbol{H}\right) \in \Re^{MN\times MN}$,
$\boldsymbol{H} = \left[\boldsymbol{H}_1^T, \boldsymbol{H}_2^T, \ldots, \boldsymbol{H}_N^T\right]^T \in \Re^{MN\times MN}$,
$\boldsymbol{B} = diag(\boldsymbol{d}) \in \Re^{MN\times N}$, $\boldsymbol{c} = \left[\boldsymbol{z}^T, \boldsymbol{z}^T, \ldots, \boldsymbol{z}^T\right]^T \in \Re^{MN}$
$\boldsymbol{f}(\boldsymbol{x}) = (f(\boldsymbol{x}_1), f(\boldsymbol{x}_2), \ldots f(\boldsymbol{x}_N))^T \in \Re^N$.

Remark 2: Following the conjecture in [16], it is convenient to choose the elements of the matrix $\boldsymbol{H}_i$ sufficiently small. In fact, given N chaotic subsystems, if the coupling is not too strong, the whole array is expected to be characterized by N positive Lyapunov exponents.

2.3. Initial and boundary conditions

Given the cell dynamics (1) and the synaptic law (2), it is not difficult to find a sets of initial and boundary conditions able to generate hyperchaotic behavior. For instance, in [16] hyperchaos is obtained by considering the initial conditions for which each cell is chaotic and by taking ring boundary conditions.

3. Chaos synchronization

Definition 2: Given two chaotic systems

$$\dot{\boldsymbol{x}} = \boldsymbol{g}(\boldsymbol{x}) \tag{5}$$
$$\dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}) \tag{6}$$

they are said to be synchronized if

$$e(t) = (y(t) - x(t)) \to 0 \text{ as } t \to 00 \quad (7)$$

where $\boldsymbol{x} \in \Re^n$, $\boldsymbol{y} \in \Re^n$, $\boldsymbol{g}: \Re^n \to \Re^n$ and e is the synchronization error [3].

In order to obtain synchronization, system (6) has to receive a proper signal s(x) from system (5). In other words, it is necessary to consider a *coupling* between the chaotic systems (5) and (6). More precisely, the following definition of *master-slave synchronization* is given [4].

Definition 3: System (5) and the dynamic system

$$\dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{y}) \quad (8)$$

where $\boldsymbol{l} \in \Re^n$ is a suitable nonlinear coupling, are said to be synchronized in master-slave configuration if $y \to \boldsymbol{x}$ as $t \to \infty$. This implies that the error system between the *master (5)* and the slave (8)

$$\begin{aligned} \dot{\boldsymbol{e}} &= \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{y}) - \boldsymbol{g}(\boldsymbol{x}) \\ &= \boldsymbol{g}(\boldsymbol{x} + e) + \boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{x} + e) - g(x) = \boldsymbol{h}(\boldsymbol{e}, t) \end{aligned} \quad (9)$$

has a (globally) asymptotically stable equilibrium point for e=O. Note that synchronization is said to be *global* if $y \to \boldsymbol{x}$ for any initial condition y(O), $\boldsymbol{x}(0)$ [5].

4. Synchronization of hyperchaotic CNNs

Now the attention is focused on the synchronization of hyperchaotic dynamics generated by high-dimensional systems. Taking into account the considerations reported in section 3, it is clear that the key problem is the choice of a suitable coupling $\boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}),\boldsymbol{y})$ able to synchronize (5) and (8).

Theorem I: Consider two hyperchaotic CNNs in a master-slave configuration

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{c} \quad (10)$$

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{y}) + \boldsymbol{c} + \boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{y}) \quad (11)$$

where the state variables have been numbered as follows:

$$\begin{aligned} \boldsymbol{x} &= \left[\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \dots, \boldsymbol{x}_N^T\right]^T \\ &= \left[x_1, x_2, \dots, x_M, x_{M+1}, x_{M+2}, \dots, x_{MN}\right]^T \end{aligned}$$

$$\begin{aligned} \boldsymbol{y} &= \left[\boldsymbol{y}_1^T, \boldsymbol{y}_2^T, \dots, \boldsymbol{y}_N^T\right]^T \\ &= \left[y_1, y_2, \dots, y_M, y_{M+1}, y_{M+2}, \dots, y_{MN}\right]^T \end{aligned}$$

Let

$$\boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{y}) = \boldsymbol{B}(\boldsymbol{s}(\boldsymbol{x}) - \boldsymbol{s}(\boldsymbol{y})) \quad (12)$$

be the nonlinear coupling, where

$$\boldsymbol{s}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{K}\boldsymbol{x} \quad (13)$$

is the synchronizing signal, with

$$\boldsymbol{K} = \begin{bmatrix} k_{1,1} & k_{1,2} & \dots & k_{1,MN} \\ k_{2,1} & k_{2,2} & \dots & k_{2,MN} \\ \dots & \dots & \dots & \dots \\ k_{N,1} & k_{N,2} & \dots & k_{N,MN} \end{bmatrix} \in \Re^{N \times MN}.$$

Then the error system (9) becomes linear and time-invariant, and can be written as

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} - \boldsymbol{B}\boldsymbol{K}\boldsymbol{e} = \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}\boldsymbol{u} \quad (14)$$

where $\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{e} \in \Re^N$ plays the role of a state feedback.

Proof By substituting equations (10)–(13) in (9), the error system becomes

$$\begin{aligned} \dot{\boldsymbol{e}} &= \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{y}) - \boldsymbol{g}(\boldsymbol{x}) \\ &= \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}f(y) + c + \boldsymbol{B}(\boldsymbol{s}(\boldsymbol{x}) - s(y)) - (Ax + Bf(x) + c) \\ &= \boldsymbol{A}\boldsymbol{e} + B(f(y) - f(x)) + \boldsymbol{B}(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{K}\boldsymbol{x} - f(y) - \boldsymbol{K}\boldsymbol{y}) \\ &= \boldsymbol{A}\boldsymbol{e} - \boldsymbol{B}\boldsymbol{K}\boldsymbol{e} = \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}\boldsymbol{u} \end{aligned}$$

This completes the proof.

Now, a theorem for synchronizing high-dimensional systems can be given. This result is based on a method recently developed in [5].

Theorem 2: A necessary and sufficient condition for the existence of a feedback gain matrix $\boldsymbol{K}$ such that the CNNs (10) and (11) are globally synchronized is that all the uncontrollable eigenvalues of the error system (14), if any, have negative real parts.

Proof. System (14) can be transformed to Kalman controllable canonical form [17] by means of a coordinate transformation e = $\boldsymbol{T}\bar{\boldsymbol{e}} = [\boldsymbol{T}_1 \quad \boldsymbol{T}_2]\bar{\boldsymbol{e}}$, with the property that $\boldsymbol{T}^{-1} = \boldsymbol{T}^T$:

$$\begin{bmatrix} \dot{\bar{e}}_c \\ \dot{\bar{e}}_{nc} \end{bmatrix} = \begin{bmatrix} T_1^T A T_1 & T_1^T A T_2 \\ 0 & T_2^T A T_2 \end{bmatrix} \begin{bmatrix} \bar{e}_c \\ \bar{e}_{nc} \end{bmatrix} + \begin{bmatrix} T_1^T B \\ 0 \end{bmatrix} u$$
$$= \begin{bmatrix} \bar{A}_c & \bar{A}_{12} \\ 0 & \bar{A}_{nc} \end{bmatrix} \begin{bmatrix} \bar{e}_c \\ \bar{e}_{nc} \end{bmatrix} + \begin{bmatrix} \bar{B}_c \\ 0 \end{bmatrix} u \qquad (15)$$

where the eigenvalues of $\bar{\boldsymbol{A}}_c$ are controllable (i.e., they can be placed anywhere by state feedback u = $-\boldsymbol{Ke}$), whereas the eigenvalues of $\bar{\boldsymbol{A}}_{nc}$ are uncontrollable (i.e., they are not affected by the introduction of any state feedback). Therefore a necessary and sufficient condition to globally asymptotically stabilize system (15) is that the eigenvalues of $\bar{\boldsymbol{A}}_{nc}$ lie in the left half plane. Since $\bar{\boldsymbol{e}} \to 0$ implies e $\to 0$, it follows that $\boldsymbol{y} \to \boldsymbol{x}, \boldsymbol{s}(\boldsymbol{y}) \to \boldsymbol{s}(\boldsymbol{x})$ and $\boldsymbol{l}(\boldsymbol{s}(\boldsymbol{x}), \boldsymbol{y}) \to \boldsymbol{0}$ ast $\to \infty$, that is, global synchronization is achieved. This completes the proof.

Remark 3: If system (14) is controllable, then all the modes can be arbitrarily assigned. As a consequence, synchronization can be achieved according to any specified feature.

Remark 4: Differently from [1], the proposed method does not require the computation of any Lyapunov exponent in order to verify synchronization. Moreover, since global synchronization is achieved, the suggested technique does not require CNN initial conditions belonging to the same basin of attraction.

5. Example

A 5-cell CNN consisting of identical Chua's circuits forming a ring is considered [16]. The cell dynamics (1) is a set of 3 differential equations with

$$\boldsymbol{C} = \begin{bmatrix} -3.2 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} 2.95 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{z} = \boldsymbol{0},$$

$$f(\boldsymbol{x}_i) = f(x_{3i-2}) = |x_{3i-2} + 1| - |x_{3i-2} - 1| , \quad i=1,\ldots,5.$$

By considering the synaptic law

$$\begin{bmatrix} I_1 \\ I_2 \\ \ldots \\ I_5 \end{bmatrix} = H \begin{vmatrix} x_1 \\ x_2 \\ \ldots \\ x_5 \end{vmatrix}$$

where

$$H = \begin{bmatrix} -H\hat{H} & 0 & 0 & 0 \\ 0 & -\hat{H} & \hat{H} & 0 & 0 \\ 0 & 0 & -\hat{H} & \hat{H} & 0 \\ 0 & 0 & 0 & -\hat{H} & \hat{H} \\ \hat{H} & 0 & 0 & 0 & -\hat{H} \end{bmatrix},$$

$$\hat{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.01 & \\ 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{0} \in \Re^{3\times 3}$$

a CNN in the form (10) is obtained, for which experimental observation of hyperchaos has been illustrated in [16]. Two projections of the hyperchaotic attractors are reported in Figs. 2 and 3, respectively. Moreover, by considering (10)-(13), the CNN master-slave configuration is derived.
Since the controllability matrix

$$|B \quad AB \quad A^2B \ldots \quad A^{14}B|$$

of system (14) is full rank, the synchronization error system is controllable [5], [17]. Therefore, Theorem 2 assures the existence of a feedback matrix $\boldsymbol{K} \in \Re^{5\times 15}$ such that y $\to \boldsymbol{x}$ *as* t $\to \infty$ for *any initial state.* For instance, the set of the error system eigenvalues becomes

$$\{-1, -1, -1, -1, -1, -2, -2, -2, -2, -2, -3, -3, -3, -3, -3\}$$

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for

$$K = \begin{bmatrix} k_1 & k_2 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 & 0 \\ 0 & 0 & k_1 & k_2 & 0 \\ 0 & 0 & 0 & k_1 & k_2 \\ k_2 & 0 & 0 & 0 & k_1 \end{bmatrix}$$

where

$$k_1 = [0.6068\ \ 0.3695\ \ 1.5547],$$
$$k_2 = [0.0034\ \ 0.0135\ \ 0.0034],$$
$$0 \in \Re^{1\times 3}.$$

Figs. 4 and 5 show the synchronization between the variables (x_4, y_4) and (x_{12}, y_{12}), respectively.

6. Conclusion

In this paper CNNs constituted by chaotic cells have been considered as a tool for generating hyperchaotic behaviors. The proposed synchronization method presents several interesting features. In particular: (1) it enables synchronization to be achieved in a *systematic* way; (2) it is *flexible*, because different complex behavior can be obtained by changing the cell dynamics or by adding new cells; (3) it does not require the computation of any Lyapunov exponent in order to verify synchronization; (4) it does not require CNN initial conditions belonging to the same basin of attraction. The proposed method has been applied to synchronize an example of hyperchaotic CNN constituted by Chua's circuits. Simulation results have confirmed the effectiveness of the suggested system theory approach.

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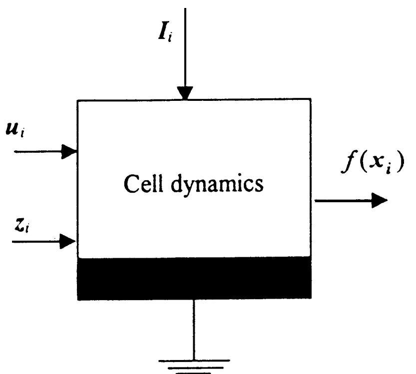

Figure 1. The basic CNN cell.

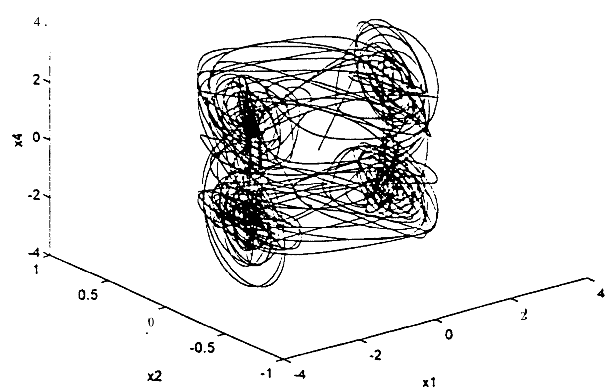

Figure 2. Projection on (x_1, x_2, x_4) of the attractor generated by the CNN.

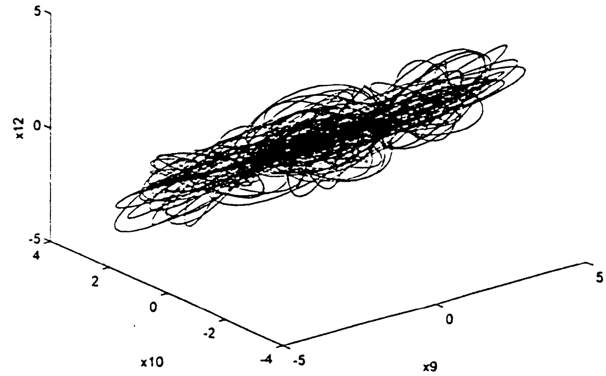

Figure 3. Projection on (x_9, x_{10}, x_{12}) of the attractor generated by the CNN.

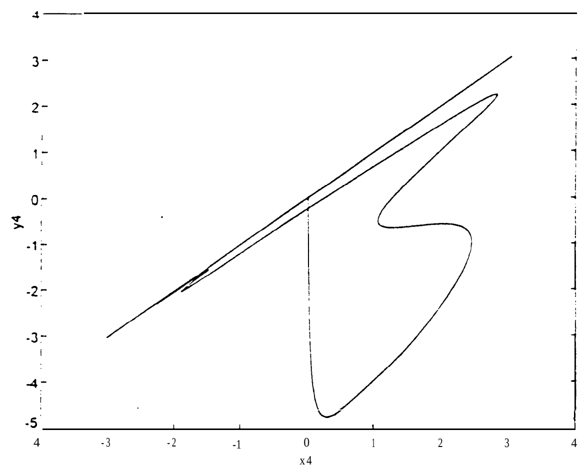

Figure 4. Synchronization between x_4 and y_4.

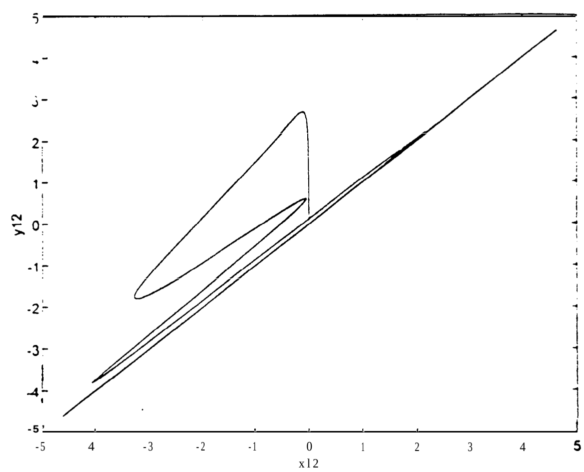

Figure 5. Synchronization between x_{12} and y_{12}.