Backstepping Design for Controlling Lorenz Chaos

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Abstract Backstepping design is proposed to control a Lorenz chaotic system. A major advantage of this method is that it has the flexibility to build the control law by avoiding cancellations of useful nonlinearities. Consequently, the goals of stabilization of chaotic motion and tracking of a reference signal are achieved with a reduced control effort. A comparison with the differential geometric method highlights the advantages of the proposed approach.

1. Introduction

Dynamic systems described by nonlinear differential equations can be strongly sensitive to initial conditions. This phenomenon is known as *deterministic chaos* just to mean that, even if the system mathematical description is deterministic, its behavior proves to be unpredictable.

Many mechanical, electrical or chemical systems can exhibit chaotic dynamics. Since chaos is unpredictable and may lead to vibrations and fatigue failures in mechanical systems, its suppression is generally advantageous. Consequently, analysis and control of chaotic systems have received ever increasing attention in recent years [1]-[11]. The pioneer work by Ott, Grebogi, and York (OGY method) [3], proposed to make a small time-dependent perturbation on one accessible parameter of the system to set the chaotic motion to one of the unstable periodic orbit embedded in a strange attractor. A drawback is that steady state solutions represent the most practical operation mode in many chaotic systems such as electronic oscillators [2] or laser systems [4]. Therefore, it is important to develop control techniques to drive a strange attractor not only to a periodic orbit but also to a steady state. In [5] a periodical external force is proposed to drive chaotic motion to a periodic orbit but not to a steady state [6]. Many feedback approaches, like conventional linear feedback in [7] and [8], have been proposed to drive the chaotic motion to a steady state. Regarding nonlinear control, in [9] a variable structure control strategy is proposed to stabilize Lorenz chaos.

A more interesting method, based on the differential geometric approach, is proposed in [6]. Since the dynamic of the original system is transformed into a linear one, linear control techniques can be used to drive the motion to the desired trajectory. However, this method has two drawbacks: 1) perfect linearization cannot be obtained because the nonlinear feedback depends on the nonlinear model of the system that necessarily contains uncertainty; 2) the resulting control law is somewhat complex and requires too much control effort because it pursues linearization of the system rather than stabilization or tracking.

In this paper, backstepping design [12] is proposed as a technique for controlling Lorenz chaos. Based on recursive application of Lyapunov's direct method, backstepping enables to drive the chaotic motion towards any desired trajectory. A major advantage of the proposed method is that it has the flexibility to choose the control law so that both the goals of stabilization and tracking are achieved with reduced control effort.

Finally, simulation results show that backstepping design requires less control effort to achieve stabilization and tracking in comparison with the differential geometric approach proposed in [6].

2. Backstepping Design for Controlling Chaos

Given a dynamic system described by nonlinear differential equations of the form $\dot{x}=f(x,u)$, where $x\in\Re^n$, a chaotic solution can exist for initial conditions *x(0)* belonging to some set $X_c\subset\Re^n$ and for control input $u\in\Re$ set to zero. From control theory perspective, controlling chaotic dynamics consists of finding a control law *u=u(x)* such that a chaotic solution *x(t)* is reduced to the desired trajectory. Backstepping offers a recursive design method for building both feedback control laws and associated Lyapunov functions. For details on backstepping design see [12].

2.1 Stabilization of Lorenz System

Lorenz system is a benchmark in studying chaotic phenomena [6], [9], [13]. It basically models convection process. It is described by the following differential equations [6]

$$\begin{aligned}\dot{x}&=-10x+10y\\ \dot{y}&=-xz-y\\ z&=xy-z-R\end{aligned}\tag{1}$$

where $R=R_0+u$ is the Rayleigh number, R_0 is the operation value, and u is the control parameter. If $R_0=28$, the uncontrolled system (i.e. $u=0$) is chaotic and there are three unstable equilibrium points: $(C_0,C_0,-1)$, $(0,0,-R_0)$, and $(-C_0,-C_0,-1)$ where $C_0=\sqrt{R_0-1}$. It is worth noting that, when the set point is the state $(C_0,C_0,-1)$, the OGY method is not applicable [6]. By translating the origin of system (1) in the set point $(C_0,C_0,-1)$, the system equations become

$$\begin{aligned}\dot{x}_1&=-10x_1+10x_2\\ \dot{x}_2&=x_1-x_2-(\sqrt{27}+x_1)x_3\\ \dot{x}_3&=\sqrt{27}(x_1+x_2)-x_3+x_1x_2-u\end{aligned}\tag{2}$$

The objective is to find a control law *u(x)* for stabilizing the state of system (2) in the origin. Since system (2) belongs to the class of strict-feedback systems, backstepping design can be applied [12]. It gives the control law

$$u=\sqrt{27}x_1-\frac{11x_1}{\sqrt{27}+x_1}-\frac{11\sqrt{27}(-10x_1+10x_2)}{\left(\sqrt{27}+x_1\right)^2}\tag{3}$$

2.3 Tracking of Lorenz System

The aim of this subsection is to find a control law for the input u such that a defined scalar output tracks any desired trajectory *r(t)*, including stable or unstable limit cycles as well as chaotic trajectories. By assuming the output $y=x_2$, let z_2 be the deviation of x_2 from the target, i.e. $z_2=x_2-r(t)$. Given $V_2=\frac{1}{2}z_2^2$, the time derivative $\dot{V}_2$ becomes negative definite by choosing the virtual control

$$x_3 = \alpha_2 = \frac{z_2 + x_1 - x_2 - \dot{r}}{\sqrt{27} + x_1}.$$

Again, by choosing $V_3 = V_2 + \frac{1}{2}z_3^2$, where $z_3 = x_3 - \alpha_2$ is the deviation of the virtual control from the stabilizing function, the time derivative $\dot{V}_3$ is made negative definite by the control input

$$u = \frac{(10x_1 - 10x_2)(\sqrt{27} + r + \dot{r})}{(\sqrt{27} + x_1)^2} + \frac{2\dot{r} + \ddot{r} - x_1 + r}{\sqrt{27} + x_1} + r(x_1 + \sqrt{27}) + \sqrt{27}x_1 \qquad (4)$$

which assures that x_2 tracks the reference signal *r(t)*.

3. Backstepping Design versus Feedback Linearization

In this section the backstepping design is compared with the differential geometric method proposed in [6] via computer simulations. The results concerning stabilization via backstepping design and feedback linearization are reported in Fig. 1, whereas the results concerning tracking are reported in Fig. 2. The control action u is switched on at $t=20$. The figures show that backstepping design requires less control effort than differential geometric method. The reason is that backstepping pursues the goals of stabilization and tracking rather than that of linearization.

4. Conclusion

Backstepping design has been applied to a Lorenz system for both stabilization and tracking. The advantages of the proposed approach can be summarized as follows: 1) it is a systematic procedure to design control laws for chaotic systems; 2) it requires less control effort in comparison with the differential geometric method.

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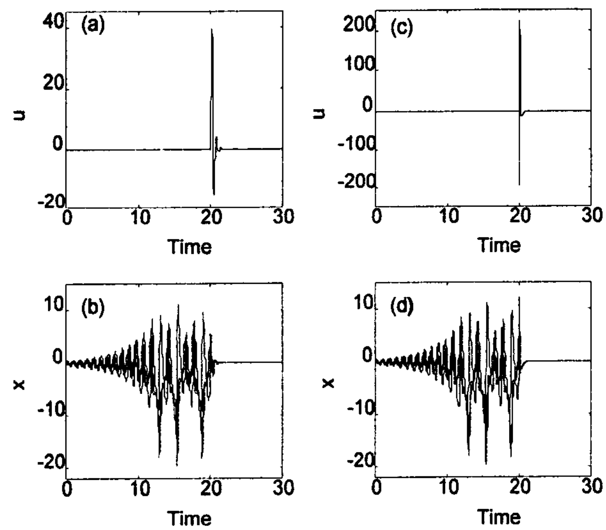

Fig. 1: Stabilization: (a) time waveform of control effort u using backstepping and (c) using feedback linearization; (b) corresponding time waveforms of x_1, x_2 and x_3 using backstepping and (d) using feedback linearization.

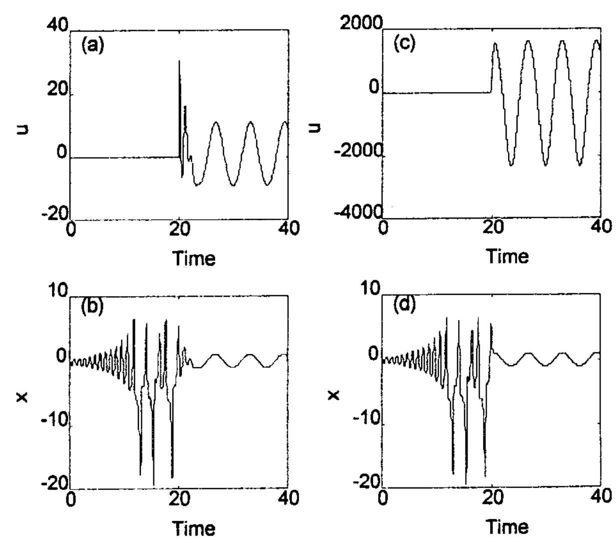

Fig. 2: Tracking of *sin(t)*: (a) time waveform of control effort u using backstepping and (c) using feedback linearization; (b) corresponding time waveform of x_2 using backstepping and (d) using feedback linearization.