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Synchronisation of hyperchaotic oscillators using a scalar signal

G. Grassi and S. Mascolo

A method is presented for synchronising two recent examples of hyperchaotic oscillators via a scalar transmitted signal. The approach, based on linear control theory, is simple and rigorous. It does not require either the computation of the Lyapunov exponents or initial conditions belonging to the same basin of attraction.

Introduction: In recent years much interest has been devoted to the study of chaotic system synchronisation [1 – 4]. This phenomenon could lead to interesting applications in secure communications [5]. To obtain higher security the adoption of hyperchaotic systems, characterised by two or more positive Lyapunov exponents, seems to be more advantageous than the use of chaotic systems with only one positive Lyapunov exponent [6, 7].

This Letter describes a technique for synchronising two recent examples of hyperchaotic oscillators [8, 9]. The approach, by exploiting results from modern linear control theory, enables synchronisation to be achieved via a scalar transmitted signal.

Synchronisation of 4D oscillators: A simple 4D oscillator containing an opamp, two LC circuits and a diode was presented in [8]. Its hyperchaotic behaviour has been confirmed by both experiment and numerical simulation. The circuit dynamics is described in dimensionless form by the following [8]:

$$\dot{x}_1 = 0.7x_1 - x_2 - x_3 \quad (1)$$

$$\dot{x}_2 = x_1 \quad (2)$$

$$\dot{x}_3 = 3(x_1 - x_4) \quad (3)$$

$$\dot{x}_4 = 3x_3 - 30(x_4 - 1)H(x_4 - 1) \quad (4)$$

$H(z)$ is the Heaviside function, i.e. $H(z < 0) = 0$ and $H(z \geq 0) = 1$.

The proposed technique consists in choosing a suitable nonlinear coupling between the drive system (eqns. 1 – 4) and the driven one:

$$\dot{y}_1 = 0.7y_1 - y_2 - y_3 \quad (5)$$

$$\dot{y}_2 = y_1 \quad (6)$$

$$\dot{y}_3 = 3(y_1 - y_4) \quad (7)$$

$$\dot{y}_4 = 3y_3 - 30(y_4 - 1)H(y_4 - 1) - 30(s(\mathbf{x}) - s(\mathbf{y})) \quad (8)$$

where the scalar transmitted signal $s(\mathbf{x})$, $\mathbf{x} \in \Re^4$,

$$s(\mathbf{x}) = (x_4 - 1)H(x_4 - 1) + \sum_{i=1}^{4} k_i x_i$$

has been designed so that the synchronisation error system is linear time-invariant:

$$\dot{e}_1 = 0.7e_1 - e_2 - e_3 \quad (9)$$

$$\dot{e}_2 = e_1 \quad (10)$$

$$\dot{e}_3 = 3(e_1 - e_4) \quad (11)$$

$$\dot{e}_4 = 3e_3 - 30u \quad (12)$$

Note that the variable $u = -\Sigma_{i=1}^{4} k_i e_i$ plays the role of state feedback. The diagram of two such coupled oscillators is shown in Fig. 1. In particular, the multiterminal black box [12] with $I = -30(s(\mathbf{x}) - s(\mathbf{y}))$ takes into account the coupling between the oscillators. Since the controllability matrix of the system of eqns. 9–12 is full rank, from linear control theory [10, 11] it follows that the system eigenvalues can be placed anywhere by proper choice of the feedback gains k_i. For instance, the eigenvalues of eqns. 9 – 12 are placed in –1 for $k_1 = -47.4927$, $k_2 = -104.1723$, $k_3 = 6.8388$ and $k_4 = -1.3567$. It is worth noting that $\mathbf{y} \to \mathbf{x}$ implies $s(\mathbf{y}) \to s(\mathbf{x})$, that is, the dynamics of the systems eqns. 1–4 and eqns. 5–8 become identical. Fig. 2 shows how the state variable $y_1(t)$ tracks $x_1(t)$ after the synchronising signal is switched on at $(t/T) = 30$, with $T = \sqrt{L_1 C_1}$, $x_1 = U_{C1}^1 / U_0$, $y_1 = U_{C1}^2 / U_0$ whereas $U_0 = 0.7$V is the forward voltage drop of the diode (see [8]).

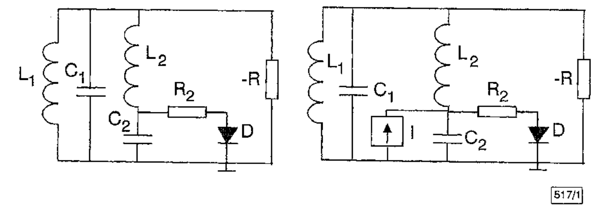

Fig. 1 *4D oscillators: synchronisation scheme*

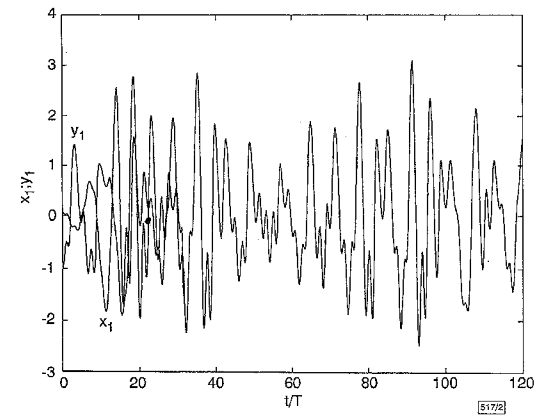

Fig. 2 *Time waveforms of chaotic variables $x_1(t)$ and $y_1(t)$ of systems in eqns. 1 – 4 and 5 – 8, respectively*

Synchronising signal is switched on at $(t/T) = 30$, $x_1 = U_{C1}^1 / U_0$, $y_1 = U_{C1}^2 / U_0$, $U_0 = 0.7$V

Synchronisation of oscillators with gyrators: Recently, a hyperchaotic oscillator realised without inductance coils has been described in [9]. It contains a negative impedance converter, two capacitors, two gyrators and a diode. By taking the circuit parameters reported in [9], the dynamics can be written as

$$\dot{x}_1 = 0.55x_1 - x_2 - 40(x_1 - x_3 - 1)H(x_1 - x_3 - 1) \quad (13)$$

$$\dot{x}_2 = x_1 \quad (14)$$

$$\dot{x}_3 = -(1/0.31)x_4 + (40/0.31)(x_1 - x_3 - 1)H(x_1 - x_3 - 1) \quad (15)$$

$$\dot{x}_4 = (1/0.33)x_3 \quad (16)$$

By considering the following driven system:

$$\dot{y}_1 = 0.55y_1 - y_2 - 40(y_1 - y_3 - 1)H(y_1 - y_3 - 1) - 40(s(\mathbf{x}) - s(\mathbf{y})) \quad (17)$$

$$\dot{y}_2 = y_1 \quad (18)$$

$$\dot{y}_3 = -(1/0.31)y_4 + (40/0.31)(y_1 - y_3 - 1)H(y_1 - y_3 - 1) + (40/0.31)(s(\mathbf{x}) - s(\mathbf{y})) \quad (19)$$

$$\dot{y}_4 = (1/0.33)y_3 \quad (20)$$

where the scalar transmitted signal

$$s(\mathbf{x}) = (x_1 - x_3 - 1)H(x_1 - x_3 - 1) + \sum_{i=1}^{4} k_i x_i$$

has been obtained by adding the current in the diode to a linear combination of the circuit state variables; a linear time-invariant error system is derived:

$$\dot{e}_1 = 0.55e_1 - e_2 - 40u \quad (21)$$

$$\dot{e}_2 = e_1 \quad (22)$$

$$\dot{e}_3 = -(1/0.31)e_4 + (40/0.31)u \quad (23)$$

$$\dot{e}_4 = (1/0.33)e_3 \quad (24)$$

Again, $u = -\Sigma_{i=1}^{4} k_i e_i$ represents a state feedback. The diagram of the synchronisation scheme is given in Fig. 3, where the black boxes with $I_1 = -40(s(\mathbf{x}) - s(\mathbf{y}))$ and $I_3 = (40/0.31)(s(\mathbf{x}) - s(\mathbf{y}))$ represent the coupling between the oscillators. Since the controllability matrix [11] of the system of eqns. 21 – 24 is full rank, the eigenvalues can be placed, for instance, in –1 for $k_1 = -0.0109$, $k_2 = 0.0178$, $k_3 = 0.0319$, $k_4 = -0.0046$, synchronising the systems of eqns. 13 – 16 and eqns. 17 – 20. Fig. 4 shows that $y_1(t)$ tracks $x_1(t)$ after the synchronising signal is switched on at $(t/T) = 30$, with $T = \sqrt{L_1C_1}$, $x_1 = U_{C1}^1/U_0$, $y_1 = U_{C1}^2/U_0$, $U_0 = 0.65$V (see [9]).

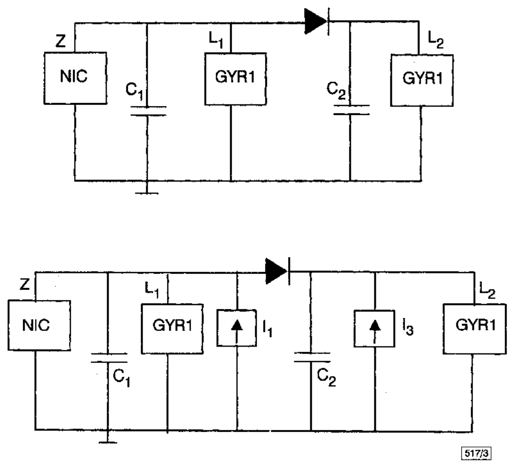

Fig. 3 *Oscillators with gyrators: synchronisation scheme*

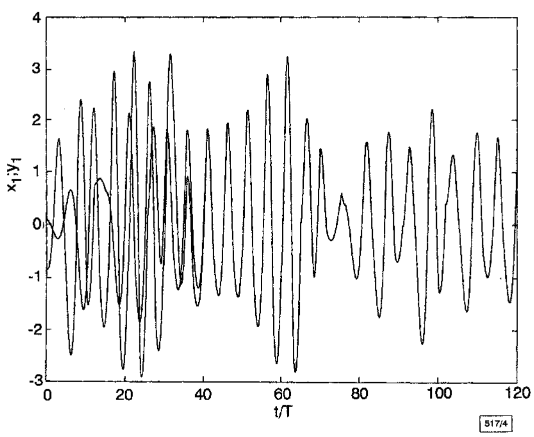

Fig. 4 *Time waveforms of chaotic variables $x_1(t)$ and $y_1(t)$ of systems in eqns. 13 – 16 and eqns. 17 – 20, respectively*

Here, $T = 0.237$ms, $x_1 = U_{C1}^1/U_0$, $y_1 = U_{C1}^2/U_0$, $U_0 = 0.65$V

It is worth noting that the error systems of eqns. 9 – 12 and eqns. 21 – 24 have been globally asymptotically stabilised at the origin. Therefore, it is not necessary for the initial conditions of the corresponding drive and response systems to belong to the same basin of attraction.

Conclusions: In this Letter a new tool for synchronising two examples of hyperchaotic oscillators using a scalar transmitted signal has been proposed. The idea is to design the synchronising signal so that a linear time-invariant error system is obtained. In this way, synchronisation can be achieved by exploiting results from modern control theory. The approach is simple, rigorous and does not require the computation of any Lyapunov exponent [7].

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Adaptive receiver for DS/CDMA communications over impulsive noise channels

Ho-Chi Hwang and Che-Ho Wei

A new adaptive algorithm is proposed for training soft-limiter based correlation receiver in which the direct sequence code division multiple access signals corrupted by impulsive symmetric α-stable noise are demodulated. The new adaptation algorithm allows simpler implementation and faster convergence speed in comparison with the traditional adaptive stochastic gradient-based algorithms.

Introduction: Adaptive linear filtering algorithms have been commonly used to train minimum mean-squared error (MMSE) linear detectors [1] for demodulating direct-sequence code division multiple access (DS/CDMA) signals over an additive white Gaussian noise (AWGN) channel. However, in some realistic communication links, some natural, as well as man-made, interference is non-Gaussian and impulsive [2]. The impulsive noise is commonly represented by the symmetric α-stable (SαS) process [2], in which the probability density function (pdf) is given by $f_\alpha(\gamma, \delta; x) = (1/2\pi)\int_{-\infty}^{\infty} \exp(j\delta\omega - \gamma|\omega|^\alpha)e^{-j\omega x}d\omega$, where $\alpha(0 < \alpha \leq 2)$ implies the 'thickness' of the tails of the pdf and the dispersion $\gamma(\gamma > 0)$ relates to the spread measure of the pdf around the location parameter, δ. When the SαS process has a smaller α, it has a largely increased probability of large amplitudes.

Because SαS noise (except α = 2) has no finite variance, the least mean p-norm (LMP) algorithm [2] and the normalised LMP algorithm [3] instead of the least-mean-square (LMS) error algorithm are used for adaptive linear filtering for SαS random processes. The value of p, constrained to $p < \alpha$, is usually taken as $p = 1$ when α is either unknown or varying in time. When $p = 1$, the LMP algorithm is called the least mean absolute deviation (LMAD) algorithm. In this Letter, a new adaptation algorithm is developed for adjusting the soft-limiter (SL) correlation receiver for demodulating the DS/CDMA signals corrupted by SαS noise.